SUBJECT: PROGRESS REPORT, JUNE 14 THROUGH JULY 11, 1954

To: Jay W. Forrester
From: Scientific and Engineering Computation Group

1. MATHEMATICS, CODING AND APPLICATIONS

1.1 Introduction

During the four week period covered by this report, 700 coded programs were run on the time allocated to the Scientific and Engineering Computation (S&EC) Group representing work that has been carried on in 40 problems.

Section 1.2 contains progress reports as submitted by the programmers together with an indication of the machine time (and, in some cases, of the programming time) expended on each problem.

The following four problems were completed during this period: #142, A Study of Shock Waves; #144, Self-consistent Molecular Orbitals; #176, Connector Provision in Automatic Telephone Exchanges; #187, Response of a Fuel-flow Controller. Various phases of problem 167 (Transient Effects in Distillation) have been completed and written up in three Master's theses for the Chemical Engineering Department.

Detailed initial descriptions are provided below for problems #185 (A Scale of Turbulence), #186 (Tracking Response Characteristics of the Human Operator), #190 (Zeeman and Stark Effect in Positronium), and #194 (An Augmented Plane Wave Method As Applied to Sodium).

Work was initiated during this period on two simulated computers to be used in the special summer session course 6.531 (Digital Computers: Business Applications). The first computer (a single address computer called SAC) is a modification of the 1953 summer session computer (described previously under problem #140) involving simulated magnetic tape and omitting the floating point programmed arithmetic. Progress on SAC will be described under problem #196.

The second computer is a 3 plus 1 address device designed to simulate an actual medium speed drum computer. It will be referred to as TAC and described under problem 197.

Development work continues on the comprehensive system of service routines (CS). Two new proposed additions to the system are described below under problem 100. The first involves the use of a so-called "director" tape which will effect the reading-in of programs in their proper sequence with the information necessary for the proper running of each program. Thus it may be possible to have such a director tape control without any human intervention a large percentage
of the S&EC scheduled machine periods. The second proposal involves a simplified form for displaying the contents of WMI storage on the oscilloscope. If this simplified form is feasible, it will reduce both the time required for the display and the number of registers needed to store the display program.

During this report period, Digital Computer Laboratory tours were conducted for three different groups of visitors. A summary of the size and affiliation of each group is given under problem 131.

1.2 Programs and Computer Operation

The Comprehensive System of Service Routines: Best, 88 hours; Combelic, 68.5 hours; Demurjian, 40 hours; Denman, 19 hours; Frankovich, 68 hours; Helwig, 90 hours; Porter, 2 hours; Saber, 22 hours; Siegel, 68 hours; WMI, 1218 minutes

The comprehensive system of service routines provides for the automatic selection and use of utility programs during computer operation. The library of utility programs is permanently recorded on magnetic tape and includes conversion programs, binary input programs, equipment checking programs, and post-mortem programs. Utility selection is accomplished by using a utility control program which has been permanently recorded on the magnetic drum.

The comprehensive system has been operating successfully during the past month.

The group 11 control program is being modified so that it can be instructed to perform particular functions by means of words appearing on a punched paper tape (called a director tape). The director tape mode is selected by putting ca0 in the right insertion register and not pressing the upper activate button. The director tape itself is placed in the mechanical reader.

In the director tape mode every transfer of computer control to the group 11 control program causes one word on the director tape to be read in. The control program then selects a particular utility program on the basis of this word and executes this utility program. The director tape vocabulary has been chosen so the director tapes, when printed out, look as much as possible like the presently used S&EC performance requests. In particular, the symbols e, ri, ric, rs, sa #
fb 100-0-0, fb 123\text{mi5}, fc 100-0-0, fc 123\text{mi5}
can be typed with their usual meaning.

A memorandum describing director tape procedure in more detail will be issued when the system is working.

Work is continuing on a modification to the post-mortem program which will display post-mortem results on the scope and on the generalized magnetic tape recording program described in M-2870.

Staff

A program for displaying the contents of CM on the oscilloscope in a simplified form is undergoing test. Each word displayed consists solely of two horizontal rows of points, with the presence of a dot indicating a binary 1
and the absence of a dot signifying a binary 0. The upper of the two rows shows the word itself; the lower row displays its complement as a check.

This program promises to be many times faster and shorter than the present Group 11 scope post-mortem, which displays the contents of CM as octal fractions. With the extension of the CSII automatic post-mortem facilities to include the scope, the need for an elaborate Group 11 program will be much lessened.

It is projected that this program, to be used only in case of emergency, will be recorded on Group 11 in place of the existing scope PM at that time, provided that tests show the display to be adequately legible and unambiguous. The additional registers made available on Group 11 will be used for needed extensions of the input program.

Siegell

101 C. Optical Properties of Thin Metal Films: Denman, 26 hours; Loeb, 1 hour; WWI, 25 minutes

A few short runs were carried out to complete the work described in previous reports. A description of the methods and results of the computations is being prepared for publication.

106 C. MIT Seismic Project: Grine, 15 hours; Simpson, 30 hours; Walsh, 10 hours; WWI, 373 minutes

The eigenroot routine mentioned in the biweekly of June 13 has been used successfully during the past period to determine linear operators which are optimum in a certain sense. These operators, used in conjunction with two other programs just written (for computing frequency characteristics of operators) and an old program, have permitted the completion of a signal-noise separation experiment begun several months ago. In this experiment, performed on controlled data, the separation depended on a relatively weak assumption about the noise-signal mixture, but compared very favorably with the results of refined filter techniques depending on almost complete knowledge of the solution. During the experiment further avenues of approach were opened up and are being studied.

A slightly different attack on the same data is also underway. In this method the multiple filter problem is being considered by examining frequency coherency between seismic trace pairs. There have been computational delays in carrying out this experiment, partly due to computational difficulties and partly to experimental results requiring procedural modification.

108 C. An Interpretive Program: WWI, 22 minutes; C. Block

The individual tapes used in developing this program have been combined into one master tape. The only work done during this period was that of making a variety of tests on this combined tape.

120 D. The Aerothermopressor: Gavril, 16 hours; Porter, 3.5 hours; WWI, 259 minutes

During the past month, considerable effort has been directed toward a revision of the numerical analysis used in the WWI program for the solution
of the first order simultaneous differential equations of the aerothermopressor process.

Recently, during computations of the aerothermopressor performance under conditions of prescribed Mach Number, curious effects were observed which could not be explained on the basis of the physical nature of the process. It was suspected that the spurious behavior was the result of accumulated error in the numerical solution, and a study of the effect of increment size was accordingly carried out on a typical case. This study indicated that truncation error was indeed a serious effect and that it was necessary to improve the numerical analysis which heretofore had been satisfactory for the constant-area process.

The initial revision consisted of a simple forward integration scheme in which $y_{n+1}$ was calculated from backward differences in $y'_n$. Four points were used with starting values determined by Euler's Method

$$y_{n+1} = y_n + h y'_n$$

The results of tests of this procedure indicated extremely serious instability in which oscillation inherent in any numerical solution of this particular set of differential equations was exaggerated by the method itself. After a detailed study of the results, it was concluded that the procedure was unsatisfactory for the following reasons:

1. The starting values obtained by Euler's Method contained sufficient error (oscillating) to invalidate the derivatives used in the difference equations.
2. Once an oscillation had begun, the forward integration procedure was unable to compensate for it since differences in $y'_{n+1}$ were not used. Since the differences tacitly involve a polynomial extrapolation of the derivative, the error was actually enhanced by the method.

A significantly more involved procedure is currently being programmed, and it is felt that no simple non-iterative method has much chance of success for the type of differential equations being treated. The new method consists of a forward and successive integration scheme and is essentially Method VII of Milne's *Numerical Solution of Differential Equations*. This procedure should not suffer from the disadvantages listed above. Tentative starting values will be determined by Euler's Method but will be adjusted by forward difference relations until the values contain no oscillation. Values of $y_{n+1}$ will be computed from difference equations involving backward differences in $y'_{n+1}$, and the calculation repeated until no change in $y_{n+1}$ is observed. It is hoped that the increased computation per increment can be partly compensated by a larger increment size.

Further computations of aerothermopressor performance are currently being postponed while the new numerical analysis is being developed.

126 C. Data Reduction: WNI, 182 minutes; D. Ross

A test run of the Mistake Diagnosis Routine operating on the Data Reduction Program was finally successful. Comparison of the results with hand calculations
showed several technical programming errors which have now been corrected. Another test run is now scheduled.

An attempt to modify the programs of Problem 171 slightly to use them as function-generating and plotting routines has been beset by logical difficulties primarily due to the length of time since the programs were first written. Once reestablished, the routines will constitute a simple Fourier synthesizer.

131. Special Problems (Staff Training, Demonstrations, etc.): WWI, 260 minutes

During the past period a total of 131 people have attended three tours of Whirlwind. On June 22, 64 members of the summer course on Operations Research were given Flexowriter and computer demonstrations and an informal short talk. Nineteen businessmen attending the course on Control Problems of the Executive received a similar tour on June 28. Thirty-two foreign students from the MIT Foreign Student Project, representing 28 countries, along with representatives from Mass. Memorial Hospital, Lincoln Laboratory, the Aeroelastic Structures Laboratory, and the Norwegian-Netherlands Atomic Energy Commission were tour guests of the laboratory on July 8.

1320. Revision, Extension and Testing of the Subroutine Library Used in Programs for Obtaining Data for the Numerically Controlled Milling Machine; routine, numerical and logical operations: Porter, 1.5 hours; Runyon, 50 hours; WWI, 78 minutes

At present, two types of programs are being run. They are concerned with milling machine tape checking and airfoil template tape preparation. Some difficulty, evidenced by the appearance of spurious error indications in the results, was encountered in checking long tapes. This seemed to be due to excessive loading on the motor of the reader since the errors disappeared when a reel which allowed the tape to turn more freely was used.

An apparently satisfactory run has been made on the wing template program. The portion which was causing trouble was bypassed by using results of a previous run.

141. Matrix Diagonalization and Inversion Subroutine: Denman, 3 hours; Mahoney, 128 hours; WWI, 158 minutes

The modification of the subroutine (developed under problem 170) for calculating the inverse, or the square root of the inverse, of a real symmetric matrix by diagonalization has been tested and is working. It has been found, however, more convenient to store the matrix elements in a different order and this change will soon be incorporated.

Some consideration is being given to the use of continued fractions in computing functions and a routine for finding \( \ln x \) is now being tested.

142 D. A Study of Shock Waves: WWI, 354 minutes

This problem was completed during this period. A terminating report will appear in Summary Report #38.
144 C. Self-Consistent Molecular Orbitals: WWI, 11 minutes

The program has been rewritten successfully and is now ready to be used. The production runs will be described under Problem 201 on the ammonia molecule.

147 C. Energy Bands in Crystals: WWI, 2311 minutes; D. Howarth

During the first week of this four-week period, all the production work which it was originally intended to carry out was completed satisfactorily. This concerned the calculation of eigenvalues at points of highest symmetry in momentum space for copper.

The results greatly exceeded expectations in that three to four figure convergence was obtained for all the states with the inclusion of between 4 and 8 terms in the secular equation. Using a larger number of terms has produced five-figure convergence in some cases. This is considerably greater accuracy than has been obtained by any other existing method for the calculation of such eigenvalues.

So encouraging were the results that it was decided to calculate also certain points of lower symmetry in momentum space; results of significant accuracy can be obtained for points of no special symmetry by solution of a 14x14 secular equation. The number of points being calculated and the fact that three separate potentials are being used have made the calculation extremely lengthy, and has been made possible only by large amounts of available computer time. The entire problem is expected to be concluded during the next bi-weekly period.

The routines used in the problem are being modified to permit of simple use for other materials, since this appears to be so powerful a method.

155 D. Synoptic Climatology: Demurjian, 3 hours; Denman, 3 hours; Blood, 15 hours; WWI, 395 minutes

Approximately 1/15th of the results have been obtained. Difficulty was encountered during this period in determining a satisfactory scale factoring arrangement. Scale factors that worked well with the first month's data (an unusually cold month) resulted in arithmetic checks when applied to the second month's data (a near-normal month). This has now been overcome. It has not been tested on the third month's data (an unusually warm month). If difficulties continue it may be advisable to rewrite the program for CSII.

159 D. Water Use in a Hydroelectric System: WWI, 785 minutes; J. D. C. Little

During the past four weeks a considerable number of interesting results have been obtained. Calculations have been performed for a variety of one to dam hydro models and a variety of criteria for optimization. Operating curves have been calculated and used for running this system under the conditions of the historical river flows. Some exploration has been made of the effect of relaxing various operating restrictions. Also the mesh size of the calculated tables has been varied somewhat to study its effect on the accuracy.
162 C. Determination of Phase Shifts from Experimental Cross-sections:
Campbell, 40 hours; WWI, 45 minutes

As part of a preliminary program to obtain plots of the sum of the squares of the percentage errors in the cross section as a function of each phase shift, one tape was successfully run giving the error as a function of $S^2_2$, the $D_{5/2}$ wave phase shift, when the $S$ and $P$ wave phase shifts have constant values. This was done at seven energies for one value of the $S$ wave phase shifts and two values of the $P$ wave phase at each energy.

166 C. Construction and Testing of a Delta-wing Flutter Model: Porter, 6.5 hours; WWI, 75 minutes; S. Gravitz

Testing and modification of the completed program is continuing. Preliminary investigations were made of two proposed lattice network models—the first to be machined from a 1/8 inch thick aluminum alloy plate, and the other to be produced from 5/32 inch plate, but otherwise geometrically similar.

For the 1/8 inch plate, using the actual proposed physical dimensions as the basis for the stiffness input to the system, reasonable values for the deflection influence coefficients were obtained. It was found that negligible changes in the stiffness input to the problem resulted in negligible changes in the deflection output, but that arbitrary input changes of substantial magnitude can cause much greater corresponding changes in the resultant output.

For the 5/32 inch thick lattice, a simultaneous program of computation on WW and construction and testing of the model is now underway. An inconsistency in the geometry input to the system was detected and the program is being modified to correct this. In the process of pinpointing this error, an independent computation program was performed by use of desk calculators and at every check point the preliminary results agreed to within an average of eight significant figures with the corresponding data obtained from the WW program. This close agreement tends to indicate that the application of Crout's Method for matrix inversion in this part of the program does not yield blown up results due to matrix ill-conditioning. Such ill-conditioning had appeared in the latter part of the program where Crout's Method is used to solve a set of simultaneous equations.

Plans for the future include determining the influence coefficients of the 5/32 inch lattice on WW and comparing the results with experimentally determined values.

167 D. Transient Effects In Distillation: Jordan, 240 hours; O'Donnell, 240 hours; Myers, 80 hours; Polk, 80 hours; Smith, 80 hours

This problem did not use any machine time during the period June 14 through July 11. This report covers the machine time used during the period May 17 through June 13. About 1/2 of the machine time used during that period was used for the problem of take-off in batch distillation. The remainder was divided about equally between the problems of equilibration in batch distillation and transients in continuous distillation.
Myers completed a Master's thesis on the second problem. Time was not available for obtaining sufficient data. However, it is expected that additional data will be taken in the future with the program which is operating successfully.

Polk and Smith completed their Master's theses on the third problem. This included considerable data and efforts to correlate it. The work is promising and Jordan is continuing it with a Master's thesis this summer. Thus far he has been working on further correlation of the data available.

Considerable effort has been made to correlate the data on take-off in batch distillation. As an aid to this effort, a program is being written for WVR which will solve the limiting case of batch distillation with no hold-up.

169. Utilizing a General Purpose Digital Computer in Switching-Circuit Design: WNI, 40 minutes; E. C. Hoy

The rule $\text{XY} + \text{X}' \text{Z} + \text{Y}' \text{Z} = \text{XY} + \text{X}' \text{Z}$ where $X$ and $X'$ are single variables (i.e. variables directly related to only one relay), $Y$ and $Z$ are products of single variables (none of $Y$ being in $Z$) has been replaced by the more general rule

$$X \left( \text{product of all variables of } A \text{ or any of the single} \right) \ast X' \left( \text{product of all or any of the single variables of } A \right) = A = X(f_1[A_1]) + X'(f_2[A_2])$$

where $X$ and $X'$ are single variables, $A$ is a product of single variables.

The runs for timing the reduction program have revealed a reduced expression that contains an incorrect term. This will be corrected.

Other methods for reducing functions have been investigated but none seem worthwhile programming in the immediate future. This is not meant to imply that the present method is better than those considered more recently, but that those considered more recently also have certain seemingly insurmountable difficulties. In fact, one of the newer methods allows easier analysis of the degree of minimization obtained. These methods will be listed in the thesis as an aid for future studies.

172 B. Overlap Integrals of Molecular and Crystal Physics: Corbato, 160 hours; WNI, 352 minutes

Since the last report, the main integral calculation programs have been partially rewritten so as to be more time-efficient. These many routines have been combined into one master tape for the convenience of future users.

Using integral values obtained from the basic subroutines, a tight-binding calculation has been carried out for graphite. These results, at present, reveal two features: 1) the wave functions and potentials need not be elaborately approximated, and 2) the non-$\Lambda$ type secular equations arising in the tight-binding method are sensitive to errors in the overlap matrix elements. The latter effect is requiring a theoretical re-examination and extension of the original problem formulation, and this work is currently in progress.
In the course of using the matrix diagonalization subroutine in the library (LSR), it was found that a considerable increase in speed was possible for medium-to high-order matrices by extensive rewriting of the program. A working version of this improved routine has been submitted to the LSR. The approximate improvement in speed is in the ratio of \((100 + 28n + \frac{1}{2}n^2)\) to \((150 + 52n + 5n^2)\), where \(n\) is the order of the matrix.

173. **Course 6-537 Digital Computer Application Practice:** WWI, 35 minutes; W. Frank

The theodolite program was run; it was properly processed through the machine but the results were all zero. It was run through once more, stopping after one complete cycle and a scope post-mortem was obtained. The program is now being de-bugged.

175 C. **Impurity Levels in Crystals:** WWI, 411 minutes; G. F. Koster

The program described previously has been altered and increased in efficiency with the help of D. J. Howarth and F. J. Corbato. This gives a saving in time of almost 40%. The inversions of the matrices \((H-E)\) are proceeding on a production basis and the computation of these inverses necessary for the study of impurity levels in nickel is almost half through.

176 B. **Connector Provision in Automatic Telephone Exchanges:** WWI, 317 minutes; B. Marrows

This problem was completed during the present report period. A final report will appear in Summary Report No. 38. The results have been included in an MS thesis presented to the Electrical Engineering Department.

180 B. **Crosscorrelation of Blast Furnace Input-Output Data:** Denman, 1 hour; WWI, 15 minutes; R. Mills

Progress on the final production program continues; the correlation and transform programs are both working, and only two short control routines remain to be checked. Although proceeding slowly, this work should be completed in the near future.

181 C. **Perturbed Coulomb Wave Functions:** H. Paul, 8 hours; WWI, 3 minutes

It has been found necessary to generate the wave-function out to \(r = 15\) in order to find the phase shift and normalization factor needed for comparison of neutron-deuteron and proton-deuteron scattering at low energies.

183 D. **Blast Response of Aircraft:** Denman, 1 hour; Sternlight 100 hours; WWI, 829 minutes

The production runs for this problem have begun, and during the past period, two hundred parameter-sets have been evaluated for aircraft blast response. A cubic function encountered in this problem is being evaluated by the Newton-Raphson iteration method, and for certain exceptional cases the iteration oscillated rather than converged. Since the normal iteration always converges from above, a check will be incorporated to stop the calculation on a given parameter and go on if the iteration starts to hunt from below. The graphs of the computer results reveal a very satisfactory, smooth blast response.
184 D. **Scattering Electrons from Hydrogen. Integral Equation**: WWI, 60 minutes; M. Newstein

The program to perform a two dimensional integral has been completed and checked. Programs to compute various integrands are being checked. Rather than compute the integrand independently at each point, advantage is being taken of the fact that the integrand varies slowly from point to point. For example, instead of computing \( f^A \) at \( x \) and \( x + \Delta x \), one uses

\[
\sqrt{A + \Delta A} \approx \sqrt{A} \left( 1 + \frac{1}{2} \frac{\Delta A}{A} - \frac{1}{8} \left( \frac{\Delta A}{A} \right)^2 \right).
\]

This procedure greatly reduces the number of operations required.

185 B. **A Scale of Turbulence**: Porter, 6 hours; WWI, 36 minutes; J. Howcroft

The correlation of eddy velocities or fluctuations between two points within a turbulent flow will generally vary inversely with the magnitude of the distance between the points. This offers a method by which eddy size may be defined, the length of the eddy being that separation of the points at which the correlation becomes insignificant.

Atmospheric turbulence covers a wide range of motions with associated length scales varying from large disturbances containing most of the energy of the motion to those smallest eddies which dissipate energy as heat by action against viscosity. J. W. Tukey states that the minimum resolution of fluctuations is fixed by the sampling interval on a stationary stochastic process. An upper limit is set by the length of the sample.

Tukey's analytic filter was applied to seven sets of wind velocity data. These data were simultaneous observations of wind speed at a height of about 2.5 meters made by four standard Hastings hot thermocouple anemometers with a time constant of 0.25 seconds. The anemometers were aligned either parallel or normal to the mean wind flow with a separation of 1, 2, and 4 meters between individual units on five of the observations, and 3, 6, and 12 meters on the remainder. The sampling periods were from 392 to 510 seconds long. The sampling interval was one second.

The analytic filter operates on the sequence

\[ x_1, x_2, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_{N-1}, x_N \]

by forming the following differences:

a. First order differences of the \( j \)-th probe,

\[ d_j^{(1)} = (x_{2j} - x_{2j-1}), \quad i = 1, 2, \ldots, \left[ N/2 \right] \]

b. Second order differences for the \( j \)-th probe,

\[ d_j^{(2)} = \left( x_{4j} - x_{4j-1} \right) - \left( x_{4j-2} + x_{4j-3} \right), \quad i = 1, 2, \ldots, \left[ N/4 \right] \]

c. Third order differences for the \( j \)-th probe,

\[ d_j^{(3)} = \left( x_{8j} + \ldots + x_{8j-3} \right) - \left( x_{8j-4} + \ldots + x_{8j-7} \right), \quad i = 1, 2, \ldots, \left[ N/8 \right] \]

where \( \left[ N/2^P \right] \) represents the integral part of \( N/2^P \).
According to Tukey, the sequences of differences as formed above provide series in which maximum spectral power is concentrated near periods of $2^p$ seconds per cycle, where $p$ is the order of the difference. The filter is only a rough one since it lacks sharp cut-off characteristics. In this problem, only the first three differences are considered.

Other investigators have shown that the sums of squares of the differences defined above are proportional to the variance and energy at their associated frequencies.

Seven parameter tapes were prepared for presentation to Whirlwind using the observed wind speed reading in tens of feet. A program was then prepared in which the differences previously defined were formed intermediately without print-out. For each difference, for each parameter tape, Whirlwind was called upon to provide the following 30 numbers as its contribution to the solution of the problem:

$$\sum_{i=1}^{[N/2^p]} d_{ij}$$

$$j = 1, 2, 3, 4$$

$$\sum_{i=1}^{[N/2^p]} d_{ij}d_{ik}$$

$$j, k = 1, 2, 3, 4$$

$$\sum_{i=1}^{[N/2^p]} d_{kj}^2d_{ik}^2$$

$$j, k = 1, 2, 3, 4$$

$$\sum_{i=1}^{[N/2^p]} d_{ij} - d_{ik}$$

$$j, k = 1, 2, 3, 4$$

where $d_{ij}$ is the $i$-th difference in the sequence of differences for the $j$-th probe. Thus 630 numbers were printed by Whirlwind.

With the results from Whirlwind, certain statistical tests were made on the data in order to ascertain the degree of validity that the $d_{ij}$ are normally distributed with mean of zero and with homogeneous variances for synoptic observations. This was accomplished by considering each difference for each of the seven sampling periods separately as a subject for the following battery of tests. First, the homogeneity of variances was tested by using the test described by Hoel for this purpose. Then an Analysis of Variance was performed to test the hypothesis that the means from each of the four probes were essentially the same, and the Student's "t" Test was applied to show that this mean was not significantly different from zero. Finally, a Chi-square Goodness of Fit test was applied to the $d_{ij}$ to determine if they could be considered as having a normal distribution with mean zero and a variance equal to the arithmetic mean of the variances of the four individual probes.
For each of the differences for each sampling period, the correlation coefficients for the six possible combinations of the probes were computed. These coefficients were plotted against distance, and the indicated length at which the coefficient becomes sensibly zero noted. Thus, an eddy with a predetermined frequency will have a length, or scale of turbulence, associated with it.

In addition to the usual correlation coefficient, the $d_{ij}$ were first squared (by Whirlwind) and then a figure analogous to the $\sum |d_{ij} - d_{ik}|$ correlation coefficient computed and plotted against probe separation. Also, the quantities $\sum |d_{ij} - d_{ik}|$ were plotted against probe separation. The last two plotted figures will be compared to the plotted correlation coefficients to determine the degree of agreement between them.

186 C. Tracking Response Characteristics of the Human Operator: WWI, 11 minutes; J. Elkind

In many control systems (fire control, missile control, aircraft flight controls, etc) the human operator is an important component of the system. To design such systems for optimum performance we must know the characteristics for all important components including the human operator.

An experiment has been completed to determine the variability in human operator tracking response characteristics. Fourier transforms of the correlation functions of stimulus and response obtained from one test with one subject have been computed by WWI. The power density and cross power density functions thus obtained were used to find a quasi-linear transfer function for the subject. These results seem promising enough and spectral density functions for other tests will be computed as soon as the required correlation functions are available.

187 C. Response of a Fuel-flow Controller: WWI, 57 minutes; M. Merwin

A system of first order nonlinear ordinary differential equations describing the response of a fuel-flow controller has been solved by M. Merwin of the MIT Dynamic Analysis and Control Laboratory on WWI making use of CS II. The solution did not prove to be physically realistic so a second problem was then solved using the same equations with different constants.

The Gill method, a variation of the fourth order Runge-Kutta technique, was used. The system was the following.

$$\dot{y} = c_1 \dot{y} + c_2$$
$$\dot{x}_3 = -c_3 \dot{y} - c_4 x_3 - c_5 x_3 - c_6 \omega_2 + c_7$$
$$\dot{x}_4 = \begin{cases} \sqrt{\omega_1 - \omega_2} & \text{if } x_3 > 0 \\ \sqrt{\omega_2} & \text{if } x_3 < 0 \end{cases}$$
\[ z = c_9 \left[ -z + c_{10} \right] \]

where
\[ \bar{k} = f(z) \]
\[ \omega_1 = \text{constant} \]
\[ \omega_2 \] is a quadratic function of \( x_4 \)

190 D. **Zeeman and Stark Effect in Positronium**: WWI, 66 minutes; H. Kendall

A program to determine the Stark and Zeeman perturbed energy levels of the transient electron-positron atom (positronium) for the \( n=2 \) group of states is underway.

Using the modified diagonalization procedure of A. Meckler the energy eigenvalues, eigenvectors, and reciprocal lifetimes of the decaying states are found approximately from the complex non-Hermitian Hamiltonian.

A variation of the present routine will soon be ready. In addition to the above information this will yield the branching ratio between optical de-excitation and direct annihilation for each energy eigenvalue.

192 D. **Frequency and Phase Spectrum Analysis of Seismograms**: WWI, 323 minutes; W. P. Walsh, 50 hours

Overlapping amplitude and phase spectra of five earthquakes were computed and plotted according to a density plot routine previously described. All of the results showed travel times of various reflected and refracted P and S waves and the dispersive characteristics of Love and Rayleigh waves for Atlantic, Pacific, and continental paths. Minimum group velocities of these waves were observed for the first time, and they should be instrumental in the determination of crystal structure.

In the next few weeks we intend to continue similar analyses on other seismograms in order to gain as much evidence as possible to corroborate claims that have already been made.

193 C. **Eigenvalue problem for Propagation of E. M. Waves**: Porter, 3.5 hours; WWI, 16 minutes; H. B. Dwight

A tape has been made for one part of the problem that normally takes about 3 hours with a calculating machine. About 10 test values have been computed with the tape. They were correct to 6 to 8 significant figures, so that it is very doubtful that there is any error in the tape. The tape is arranged at present to handle 15 of these problems at once, and as a large number of solutions are needed, these are now being asked for in a routine manner by Lincoln Laboratory.

A program for a second part of Problem # 193 has been typed and submitted, No. 193-73-3. Its length is about equal to that of the first part. It will, as now planned, ultimately be put in series with the first part.
Many properties of solids can be explained successfully (conduction, semiconduction, specific heat, magnetic susceptibility, etc.) if the energy levels of an electron in the solid can be obtained. For simplicity workers have assumed the electrons to move, as a first approximation, independently of one another. This approximation is known as the band or one-electron approximation. Even so, solutions are extremely difficult. In the past three methods have been used to obtain solutions, each method characterized by its particular approximation. They are: the Bloch method (method of tight-binding), the plane-wave method, and the cellular method (method of Wigner-Seitz and Slater). More recently the method of orthogonalized plane waves has been applied to the problem of solids. Last year the method of augmented plane-waves was proposed. This method embodies the better features of all the previous methods. Howarth who has recently programmed the method on WWI has found the method which is simple mathematically but extremely tedious, ideal for machine calculation.

In applying the method to sodium the potential of an electron in sodium crystal is represented in a unit cell by a spherically symmetric potential bounded by a sphere. At the center of the cell of this sphere there is joined continuously a constant potential which extends throughout the remainder of the cell. The approximate solutions are taken to be, within the sphere, solutions of the radial Schrödinger equation for the potential and a particular energy $E$, say; without the sphere they are taken to be a plane wave. The radial solution is joined continuously to the plane wave. The energy $E$ is then determined as a value which minimizes the expectation of energy of the solution. These wave functions form a set characterized by an energy $E$ and a propagation vector of its plane wave part. A set of these wave functions are then combined according to combinations which satisfy a standard non-lambda type secular problem of quantum mechanical perturbation theory. The set—and so the order of the secular matrix—is increased until a succeeding addition does not alter the lowest eigenvalue within a fixed amount. This eigenvalue is taken to represent a solution of the problem.

Howarth has already programmed this method (Problem 147) and has applied it to copper crystal. Details of his programming can be found in the various bi-weekly and quarterly reports. Besides applying the program as it stands to sodium, a modified augmented plane-wave method will be programmed. Because it is basically the same method it will utilize Howarth's program almost totally. The reason for the modification is that in future applications of the method higher energy levels of more intractable solids will be desired and though Howarth has found for the lower levels of copper that the method is extremely successful, the higher levels can be obtained as accurately as the lower levels only at the expense of an increasingly larger secular matrix which results in an increasingly larger amount of machine time per solution.

The following problems used computer time but did not report:

107 C. (a) Autocorrelation and (b) Fourier Transform; Evaluate Integrals 119 minutes

122 B. Coulomb Wave Functions 15 minutes
197 Three Address Computer

1.3 Operating Statistics

1.3.1 Computer Time

The following indicates the distribution of WWI time allocated to the S&EC Group during the four week period covered by this report.

<table>
<thead>
<tr>
<th>Programs</th>
<th>155 hours, 8 minutes</th>
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<tbody>
<tr>
<td>Conversion</td>
<td>11 hours, 17 minutes</td>
</tr>
<tr>
<td>Magnetic Drum Test</td>
<td>1 hour, 34 minutes</td>
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<tr>
<td>Magnetic Tape Test</td>
<td>1 hour, 50 minutes</td>
</tr>
<tr>
<td>Scope Calibration</td>
<td>1 hour, 22 minutes</td>
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<tr>
<td>PETR Test</td>
<td>2 minutes</td>
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<tr>
<td>Demonstrations (#131)</td>
<td>4 hours, 20 minutes</td>
</tr>
<tr>
<td>Total Time Used</td>
<td>175 hours, 33 minutes</td>
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<tr>
<td>Total Time Assigned</td>
<td>183 hours, 10 minutes</td>
</tr>
<tr>
<td>Usable Time, Percentage</td>
<td>95.9%</td>
</tr>
<tr>
<td>Number of Programs</td>
<td>700</td>
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</table>
2. COMPUTER ENGINEERING

(K. McVicar)

Computer reliability increased during the past biweekly period. This was, no doubt, partially because of additional precautionary measures being taken by those installing and changing terminal equipment to make sure that interference with WWI operation is minimized.

A program is under way to simplify and standardize maintenance and trouble-shooting routines for the computer and terminal equipment. An aim of this undertaking is the elimination of the need for highly-specialized personnel in the diagnosis of the majority of failure types. Efforts are being concentrated on marginal-checking techniques and programs which simplify trouble location. Twice weekly classes are being held to increase the dissemination of system logic and circuitry among the system technicians. It is felt that an educational program for the terminal equipment is especially desirable since this field has been in the past the exclusive province of a few.

2.1 WWI System Operation

(L. L. Holmes, A. J. Roberts)

During the past biweekly period the computer dependability was excellent.

We have renewed our efforts to standardize the circuitry of the arithmetic-element control. The step-counter output panel has been eliminated. The step counter now uses the bus-driver circuits in A row. The add-to-step-counter circuitry was modified and resulted in a considerable improvement in the step-counter panel's margins. Timing checks have been recorded for the division operation. The divide-control panel will be extensively modified. Included in the changes will be the exclusion of approximately 11 tubes of the panel's pulse-distribution section.

A considerable amount of the installation-day work has been dedicated to the revision of the marginal-checking logic of the in-out-control system. The work will require about 6 more installation days. These changes, in addition to the recent introduction of several new IOC marginal-checking-program routines, will simplify the detection and analysis of troubles with this section of the computer.

(R. H. Gould)

The timing register on real-time clock has been made more versatile. Another source of pulses (which must be synchronized with the computer "START DELAY" pulses) may be substituted for the 60 pps by feeding the pulses into J1-1 on the remote-video-switch panel (RVSP) in E7 and throwing ON the toggle switch on this panel labeled "EXTRAORDINARY INPUT." The first two flip-flops of the counter which cannot be read out
may be bypassed to give a time count with a 1/60-second increment or to count all of the pulses fed into J1-1 of the RVSP. This is done by throwing ON the toggle switch on the RVSP labeled "BY-PASS 2 PRECOUNTER". Each switch has an indicator light in test control which is lit when the switch is ON. In normal operation the switches are OFF and the lights are OUT.

2.2 Terminal Equipment

2.21 Magnetic Drums (H. L. Ziegler)

Changeover of the auxiliary drum to electronic head switching for writing is now complete. The change-group delay will soon be reduced to a few microseconds, the exact value to be determined by tests now being conducted. If anyone has reasons for wanting the present 32-msec. delay retained temporarily, he should contact Ken McVicar or Bob Gould immediately.

Plans for the drum-chassis test setup are essentially complete, and the sheet-metal portion of the job should be nearly finished by 9 July.

3. ADMINISTRATION AND PERSONNEL

New Staff (J. C. Proctor)

Arthur Goddard is working as a DIC Staff member in Group 6345.

Staff Terminations

Sylvio Desjardins
Christopher Laspina
Hubert Henegar
Edwin Kopley
Basil Remis
Guy Young

New Non-Staff (R. A. Osborne)

Ruth Braid is a new messenger girl at the Barta Building.

Transferred Non-Staff

Jean Friberg has been transferred to the Travel Office of Division 1.

Nora McNeil has been transferred to Buildings and Power.

Mary Sexton has been transferred to the Director's Office, where she is working in the Library.
Distribution List for S & EC, or Group 6345

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