SUBJECT: BIWEEKLY REPORT, MARCH 21, 1954

To: Jay W. Forrester

From: Scientific and Engineering Computation Group

1. MATHEMATICS, CODING AND APPLICATIONS

1.1 Introduction

During the period covered by this report 334 coded programs were run on the time allocated to the Scientific and Engineering Computation (S&EC) Group. These programs represent part of the work that has been carried on in 32 of the problems that have been accepted by the S&EC Group. Progress on 20 of these problems is given below in terms of programming hours, minutes of computer time, and progress reports as submitted by the programmers in question.

Two new problems were initiated during this period. Problem #171 will investigate modifications to existing techniques for the numerical calculation of Fourier cosine transforms to obtain improved power spectrum estimates. This investigation is being carried out by D.T. Ross in partial fulfillment of the requirements for an M.S. degree in the Electrical Engineering Department. In problem #172, F.J. Corbató of the MIT Physics Department is developing routines for the evaluation of two-center overlap integrals that arise in molecular and crystal physics.

H.B. Frost has completed the calculations he needed for his study (#152) of diffusion in an oxide coated cathode. A summary of this problem will appear in Whirlwind Summary Report No. 37. The results will be included in Mr. Frost's doctoral thesis to be submitted to the MIT Electrical Engineering Department.

Intensive work has continued on the revision of the Comprehensive System of Service Routines (CS II). A brief internal memo (Bulletin Board Memo #55) has been circulated defining CS II and indicating how it differs from CS I. A detailed description of these changes will soon be available in Digital Computer Laboratory Memorandum M-2741. Use of CS II has already begun. However, it is not expected to be used on a routine basis until after April 1, 1954.
1.2 Programs and Computer Operation

The following summary is included as a guide for interpreting the abbreviations used below. A more detailed description of the terms involved can be found in M-2497.

a. The upper case letter following the problem number has the following significance:
   - A implies the problem is NOT for academic credit, is unsponsored.
   - B implies the problem is for academic credit, is unsponsored.
   - C implies the problem is NOT for academic credit, IS sponsored.
   - D implies the problem is for academic credit, IS sponsored.

The absence of a letter indicates that it is an internal S&EC problem.

b. DIC denotes the Division of Industrial Cooperation.
   DCL denotes the Digital Computer Laboratory.
   CMMC denotes the Committee on Machine Methods of Computation.
   DDL denotes the Division of Defense Laboratories.

100. Comprehensive System of Service Routines, developed by the S&EC Group at the Digital Computer Laboratory for the input conversion of suitably prepared punched paper tapes. When so requested, these routines automatically provide a program with suitable programmed arithmetic, cycle-counting, and output facilities.

   DGL Staff: Arden, 26 hours; Combelic, 65 hours; Demurjian, 42.25 hours; Denman, 82 hours; Frankovich, 45 hours; Helwig, 40 hours; Kopley, 8 hours; Porter, 18 hours; Siegel, 56 hours; WNI, 1,561 minutes

   Testing and modification of the expanded system of utility programs has been continuing.

   The input program, the summer session program and the various test programs appear to be working correctly.

   The CS II programs are being modified so that the programs in the CS are called in from the drum instead of magnetic tape unit 0. The programs appear to work correctly up to the third pass. The output adaptation programs and the output blocks will be added after the program has been successfully tested.

   Testing of the post-mortem program continues.

   Staff

   The automatic post-mortem program for CS II has been undergoing test. Several minor errors in the program have been detected and will be corrected.

   Some difficulty was encountered with the buffer drum, which was not operating properly. This drum malfunction caused several correctly written programs to fail to work and resulted in the loss of considerable computer time.

   Siegel
Generalized decimal numbers have been included as a possible automatic post-mortem mode and a programmed arithmetic post-mortem is being tested in preparation for inclusion as an automatic feature. Testing of these and other minor changes is continuing.

Arden

A new PA Post-Mortem (PA PM) program has been written for CS II. It forms a part of the new system of post-mortem programs to be stored on Magnetic Tape Unit 0 and semi-permanently on the buffer drum. In response to a request for a PA PM on the Manual Intervention Register (MIR), this PA PM program examines the stopped program (which has been stored on group 0 of the auxiliary drum) for the presence of a PA. If it is a CS I PA, the CS I PA PM is given (the CS I PA PM program is being rewritten for this purpose by D. Combelic), and control returns to the main PM control program. If neither a CS I nor CS II PA appears to be in the stopped program, "no PA" is printed out on the indicated output unit. If a CS II PA is found, the program continues with the CS II PA PM.

If this is a tape PM request, the program again looks for a PA, and if it finds a CSI PA, gives this PA PM. If neither PA is found, the program searches the requests appearing on the tape, and if either interpreted instructions or generalized decimal numbers have been requested somewhere on the request tape, "no PA" is again printed out. If neither of these types of requests is found, nothing is printed. In either case, the program then continues to give the PA's requested on the tape.

If a PA PM is requested via the MIR or is initiated by a tape request, and a CS II PA is found, but has not been used (the CS II PA is being modified so that it can give this information), the words "PA unused" are printed, and control goes back to the main PM control program.

If a CS II PA is present and used, the PA PM program prints out the number system used in the PA, a title, where the program stopped, what instruction was being performed when the program stopped, the contents of the register referred to by this instruction (if this is significant), the contents of the MRA, the absolute address of the first register of the first buffer used and the numbers and contents of all the buffers (if no buffers are used, this is omitted), similarly for the cycle-counters, and the jump table (the last 5 transfers of control by icp or isp instructions).

A typical CSII PA PM print might appear as follows:

(24,6) PA PM
stopped at 105 |105|its300 |300|+.123456789|+6 MRA|+.123456789|+32
1765|b|+.123456789|+12 1b|+.123456789|+37
1771|0|0.5 |1|2.7 |2|0.0 |3|4.11 |4|1*|3|5|2.6
57|isp|64 |66|icp|41 |52|icp|54 |57|isp|64 |77|icp|92

* The three vertical bars(///) after a counter indicate that this was the last counter referred to by the program.

Denman
The output blocks for CS II were rewritten to permit changing the location in storage of the block length table. The original location would have limited the number of output requests that could be processed by the output adaptation.

In CS II the request "MOA" will record a decimal point on the delayed printer. The stop code, for which this request was used in CS I, will be handled by two new special requests. They are "MOA end" for regular WWI mode and "iMOA end" when exit is desired in the interpreted mode. "MOA end" and "iMOA end" record successively on magnetic tape a shift to lower case, stop code, two carriage returns and another stop code character. These requests should be used to mark the end of recording on magnetic tape output.

Demurjian

Step 3 of the Output Adaptation for CS II has been translated into an optimum-length program by removing provisions which are no longer required.

Kopley

The final two tests for the first step of the Output Adaptation were run successfully. This program is now ready for inclusion in CS II.

The fourth step of the Output Adaptation has been rewritten and is now ready to be tested. This step has been revised so that it will work with either direct or indirect conversion.

Porter

101 C. Optical Properties of Thin Metal Films on transparent backings are determined and printed out automatically by this program; the input data consist of the observed reflection and transmission coefficients, the index of the backing, the wavelength, and the sample thickness. The program calculates by means of an iterative procedure and prints out the index of refraction and the absorption coefficient of the film, the rate of variation of these constants with reflection and transmission, and the film's conductivity and dielectric constant.

:for Professor L. Harris, Chemistry Department, Dr. A.L. Loeb
:by Dr. A.L. Loeb(DIC), 15 hours; Richmond, 30 hours
:DOL Staff: Deman, 1 hour; WWI, 23 minutes

In the last biweekly report equations were mentioned which enable one to obtain a first estimate of the optical and electrical constants of thin metal films for infrared radiation. These equations have been combined with the program computing reflection and transmission in terms of the optical constants to check the validity of the approximate equations. Thus a set of $R$ and $T$ is taken to begin with, and the optical constants are calculated approximately. These constants are then stored in the "Main Program" for calculation of the corresponding $R$ and $T$. If the difference between the calculated and observed values is less than the expected experimental error, the approximation is considered satisfactory. The entire set of data reported by Waltersdorff has been run in this manner in about six minutes, and the results showed that the approximate equations are good to excellent.
106 C. MIT Seismic Project is concerned with the development of methods for locating deep reflections from underground strata in seismic prospecting. The basic method is one of prediction by means of an optimum linear operator.

:for Professor P.M.Hurley; Geology and Geophysics; Professor G. Wadsworth, Mathematics Department
:by E.A.Robinson(Res.Assoc.); Briscoe, 36 hours; Simpson, 20 hours; Walsh, 25 hours
:DCL: WWI, 357 minutes

Recent work on problem 106 has been concentrated on the computation of frequency information from seismograms, in the form of either power spectra or Fourier amplitudes and phases. This information is necessary for the consideration of certain new approaches to linear operator selection. In one program a density plot scope display is used as an output.

108 C. An Interpretive Program is being developed that will accept algebraic equations, differential equations, etc. expressed on Flexowriter punched paper tape in ordinary mathematical notation (within certain limits imposed by the Flexowriter) as input and automatically provide the desired solution.

:for Dr. J.H.Laning, Jr., Instrumentation Laboratory
:by Dr. J.H.Laning, Jr.(DIC),10 hours; Zierler, 40 hours; Block,45 hours
:DCL: WWI, 156 minutes

The runs during the past two weeks have been primarily testing problems. The function routines P13-P20 (see Laning and Zierler, Instrumentation Laboratory Report E-364) have been tested and work. An attempt to use the Photoelectric Tape Reader instead of the mechanical tape reader to read in the problem tape (line by line) has not been successful.

A problem for T.Y.Toong (see M-2699 Biweekly - February 21, 1954), after a programmer's error had been corrected, ran successfully giving very satisfactory data.

113 C. A Stress Analysis of an L-shaped Homogeneous Planar Structure is being made for the case of a concentrated static load. This structure is approximated by a framework of bars which will deform in the same manner as the prototype. This framework is then analyzed using the principles of virtual work and Southwell relaxation techniques. Boundary conditions have been specified for the edge of the framework so that the deformations of the model will conform to the actual deformations of the structure.

:for Professor J.S.Archery, Department of Civil and Sanitary Eng.
:by S. Sydney(Res. Assist. CMMC), 35 hours
:DCL: WWI, 186 minutes

An analysis has been made for two load conditions on the "coarse" grid. Stress patterns have been obtained for these two cases, and these results will be inserted into the "fine" grid as initial conditions. This technique may reduce the problem of convergence since the initial data will be closer to the final results.
C. **Spherical Wave Propagation** produced by the sudden release of a spherical distribution of compressed air in the atmosphere is being studied by numerical means. This involves replacing a set of non-linear hyperbolic partial differential equations in 2 independent and 2 dependent variables by a set of difference equations written along characteristics. An iterative procedure is used to solve these equations.

For Professor C.C. Lin, Mathematics Department
By A. Ralston (CMMC), 15 hours
DCL: WWI, 252 minutes

Further trouble in the past two weeks was caused by the nonconvergence of the numerical process at one point. This caused the program to get into a loop. This trouble has now been corrected and a production run is scheduled for early in the next biweekly period.

D. **The Aerothermopressor.** This problem is concerned with the development of a device for increasing the stagnation pressure of a hot, high velocity gas stream by means of evaporative cooling. The analytical investigation being carried out on WWI involves the step-by-step solution of seven simultaneous non-linear differential equations which describe the thermodynamic and dynamic behavior of the compressible flow within the Aerothermopressor.

For Professor A.H. Shapiro, Department of Mechanical Engineering, DIC 5-6985, ONR N501-07878
By B.D. Gavril (DIC), 10 hours
DCL: WWI, 111 minutes

Computations relative to the thermodynamic performance characteristics of the aerothermopressor were resumed during the current biweekly period. These computations were concerned with improving performance by suitable cross-sectional area variation and were carried out in a routine manner with the existing aerothermopressor WWI program.

An addition to the program was made which enabled the entropy of the flow to be computed and printed with the other properties of the flow. Due to the relative length of this subroutine, it was necessary to use the auxiliary magnetic drum.

As time permits, it is planned to revise the aerothermopressor program while at the same time taking advantage of CS II. The current program involves the one bank FA and was written in April, 1953.

B. **Coulomb Wave Functions.** Regular and irregular solutions of the radial Schrödinger equation with a coulomb potential are being sought. These solutions must have the proper asymptotic form to correspond to scattered waves from such a potential. The solutions are written as a power series in the radial variable and the coefficients are determined by recursion formulae.

For Professor H. Feshbach, Professor P.M. Morse, Physics Department
By A. Temkin (Res. Assist. CMMC), 25 hours
DCL: Staff; Siegel, 3 hours; WWI, 19 minutes

A test program has been written and successfully tested to solve the recursion relation:
\[(n-L-1)(n+L)a_n^L = 2\eta a_{n-1}^L - a_{n-2}\]

where \(L\) and \(\eta\) are parameters and \(n\) (which starts at \(-L\)) is to run until the coefficients \(a_n^L\) get less than a preassigned amount. These coefficients will be eventually needed in the power series expansion of the Coulomb wave functions.

123 C. Earth Resistivity measurements are used to calculate the Slichter kernel function which, in special cases, can be analyzed to give the actual distribution of resistivity. The method involves least-square fitting a set of polynomials to the measured surface-potential function and integrating the product of this set and the zero-order Bessel function.

:for P.M.Hurley, Department of Geology and Geophysics,DIC 5-6915
:by K. Vozoff(Res.Assist.CMMO), 50 hours
:DDL Staff: Demurjian, 1 hour; WWI, 21 minutes

The program for calculating the Slichter kernel integral by iteration was completed, but the integration took far too much time. A shorter method of doing it was suggested by T.R.Madden, and this was analyzed and programmed. The new method is similar to the old one in that it first fits a set of Legendre polynomials to the original potential data. The integral is then of the form

\[k(\lambda) = \lambda \int_0^\infty r \psi(r) J_0(\lambda r) \, dr + \lambda \left[ r \psi_c \int_0^\infty J_0(\lambda r) \, dr \right]\]

\[= \lambda \int_0^\infty Q_0^c(r) J_0(\lambda r) \, dr + \lambda \left[ r \psi_c \int_0^\infty J_0(\lambda r) \, dr \right]\]

\[= \int_0^\infty Q_0^c(x) J_0(x) \, dx + \left[ r \psi_c \int_0^\infty J_0(x) \, dx \right]\]

where \(x = \lambda r\)

\[a = \lambda r_c \]

\[\left[ r \psi_c \right] = r_c \psi(r_c) = K\]

\[k(\lambda) = \int_0^\infty \left[ c_0 + c_1 x + c_2 x^2 + \ldots + c_5 x^5 \right] J_0(x) \, dx + K \int_0^\infty J_0(x) \, dx\]

The integrals

\[\int_0^\infty x^n J_0(x) \, dx\]

can be evaluated analytically. For \(n\) odd, the integral appears in closed form. For \(n\) even, the integral contains the sum of an infinite number of terms

\[\left[ J_{n+1}(a) + J_{n+3}(a) + \ldots + J_{n+1+2l}(a) + \ldots \right]\]
In the range of values of $a$ considered, this sum converges so that terms beyond $J_{30}(a)$ can be neglected.

The remainder term,

$$K \int_{0}^{\infty} J_{0}(x) \, dx$$

is evaluated in the same manner ($n = 0$).

The resulting expression is a sum of the form

$$k(\lambda) = \sum_{i=0}^{30} d_{i} J_{i}(a)$$

to be evaluated for each value of $\lambda$ and the corresponding value of $a$. The values $J_{0}(a)$ and $J_{1}(a)$ are calculated using the first 12 terms in the series expansion, which gives an accuracy of $10^{-5}$ in this range of $a$. The following terms $J_{2}$, $J_{3}$, etc., are calculated using the recursion expression for them.

One error in the program has been found and corrected. Further tests have been delayed by conversion difficulties.

The first half of the program for the analysis of the kernel function has been tested. An error has been found at the end of this program and has been corrected.

126 C. A Data Reduction Program for use in the Servomechanisms Laboratory is being developed in separate stages to be combined at a later date. The first stage is concerned with devising a program to fit polynomials to arbitrary empirical functions using a least squared error criterion. The procedure makes use of Legendre polynomials and matrix multiplication.

:for J.E.Ward, Servo Laboratory, DIC No. 7138, AF33(616)2038
:by D.T.Ross(DIC), 5 hours; Turyn, 40 hours; Hamilton, 60 hours
:DCL: WWI, 66 minutes

Tests run on the Polynomial Fit program seem to indicate that smoothness of differences of data does not affect the degree of the optimum approximating polynomial. For example, the optimum polynomial for a section of the sine function was of degree 11; it was not changed very much when the sine put in was accurate to only 5 digits instead of 8.

It was discovered that poor runs of part 1 of the Data Reduction problem were due to a numerical bias in the data which had not been removed. The program has been changed to correct this.

The scales on some of the scope plots in the Polynomial Fit program are being changed.

132 C. Subroutines for the Numerically Controlled Milling Machine are being revised and tested. The set of subroutines facilitates programming of the computations involved in the preparation of numerical data used to control the milling machine. The subroutines involve routine numerical and logical operations.

:for J.O.McDonough, Servo Laboratory, DIC No. 6873
:by J.H.Runyon(Res.Assist. E.E.), 20 hours
:DCL: WWI, 57 minutes
A subroutine for determining cut spacing across a particular cross-section for constant scallop height was successfully tested.

A series of milling machine tapes for a conical surface is being prepared. These illustrate two different methods of cut spacing and two degrees of approximation to the surface. If the results of this investigation are satisfactory, tapes will be prepared for a conic wing section.

149 C. Digital Methods of Detecting Signal from Noise are being investigated. A sequence of binary numbers will simulate the message wherein regions of high density of ones are signal regions and those with low density of ones are noise regions. Various methods of detecting the change from one region to another, as well as the length and midpoint of the signal regions are being studied.

:for J.V.Harrington, Lincoln Laboratory
:by G.P.Dinneen, Lincoln Laboratory, 20 hours; Reed, 2 hours
:DCL: WWI, 106 minutes

The second phase of this study has been started. A successful run was made to determine the distribution of ones in the generated intervals corresponding to noise and corresponding to signal. Emphasis during this phase will be on the modified success run detector.

Some difficulty, due to a programmer's error, has been encountered in the calculation of the mean and variance of the locations of the center of the signal regions.

152 D. Diffusion in an Oxide-Coated Cathode is a program to calculate the effects of combined thermal and electrolytic diffusion that occur in an oxide-coated cathode when current is caused to flow through the cathode.

:for W.B.Nottingham, Physics Department, DIC No. 6345
:by H.B.Frost(Res. Assist, E.E.), .5 hour
:DCL: WWI, 66 minutes

During the past period two runs were completed successfully. One of these runs was the completion of an earlier run, while the other run was the calculation of a complete decay and recovery characteristic, with variable current, constant field conditions. These runs complete this problem as it is now envisioned, although it is possible that one or more additional runs may be required if the calculated data prove insufficient in range for the necessary experimental comparisons.

155 B. Synoptic Climatology. A multiple regression formula is used to predict temperatures from pressure distributions described by Tschebycheff polynomials. The matrix of scalar products which is used in the calculation of the coefficients of the multiple-regression system is being calculated on WWI.

:for Professor T.F.Malone, Meteorology Department
:by R. Miller(DIC), 40 hours; Friedman, 40 hours
:DCL Staff: Demurjian, 2.75 hours; Denman, 3 hours; Porter, 6.5 hours; WWI, 419 minutes

A cross-product program was successfully tested. This program has been used to obtain the cross-products of the orthogonal polynomial coefficients with the temperature at five localities and the precipitation over three river drainage (continued on page 9)
basins. Four sets of orthogonal polynomial coefficients were used denoting, respectively, the sea-level pressure surface, and the upper-air pressure surface using height, probability of height, and standardized height units. Friedman

Specification of the surface pressure pattern for half-a-hemisphere has been completed for the months of January 1948-1952. The sums and cross-products necessary in a 52 x 52 matrix used to predict surface pressure values for a major portion of the North American Continent twenty-four hours in advance have been completed. Solution of the matrix is now being performed by desk calculator using the Crout method. Miller

161 C. Response of Mass-Plastic Spring System to Transient Loading: 2nd order non-linear difference equation representing the response of building foundations to transient shock loads is being studied in order to develop criteria for the design of blast resistant foundations. The footing is represented by a concentrated mass, and the soil by a variable mass and an elastic-plastic spring. A Runge-Kutta fourth order integration procedure will be used.

:for Professor R. Whitman, Asst. Prof. of Soil Mechanics
:by S. Sydney(Res. Asst. CMMC), 25 hours
:DCL Staff: Kopley, 1 hour; WII, 36 minutes

This program, prepared for an analysis of cohesionless soils, such as sand and gravel, is operating satisfactorily. Some results have been obtained and have been checked by hand calculations. Production runs will be made and analyzed for this case before the work on cohesive soils such as clays is started.

163 C. Ferrite Phase Shifters in Rectangular Wave Guide; transcendental equation. The electromagnetic boundary value problem dealing with the non-reciprocal ferrite phase shifter in rectangular wave guide has been solved. Special cases (assuming negligible magnetic losses) of the resulting complicated transcendental equation have been computed by hand. Additional computations by machine are required to investigate other ferrite materials and to establish a frequency dependence. Since magnetic loss is a figure of merit for the system described it is essential that some investigation be made. For cases in which the losses are significant the system will be a non-reciprocal ferrite attenuator. The numerical solution will be obtained by operating on two simultaneous transcendental equations.

:for Dr. Benjamin Lax (DDL)
:by K.J.Button(DDL), 60 hours
:DCL Staff: Demurjian, 1 hour; WII, 28 minutes

The fourth routine for the first part of the problem was run using parameters corresponding to a new ferrite and a different frequency that is now being used by the experimenters. A plot of the results revealed a small discontinuity of the first kind (finite jump) after about 2/3 of the program had been performed. When a detailed analysis failed to detect any error in the program, a fifth run was attempted. This fifth performance ran correctly and did not reproduce the discontinuity. A tape was prepared which doubled the thickness of the ferrite and a sixth performance was requested.
A second test performance was submitted for the second part of the problem. The program parameters printed out for purposes of analysis were hand calculated and found to be correct. The end results were of the proper order of magnitude. Since the test program was judged to be correct, additional routines were added to increase the accuracy and quantity of the results. The third performance printed results that appear to be of the correct order of magnitude. This performance utilized parameters corresponding to the new ferrite and frequency described in Part I.

Future Plans: Since the sixth performance of Part I (just received) has been found to be correct, additional tapes will be made to change parameters for the investigation of resonance conditions, frequency band widths, new ferrites and new frequency bands. The same conditions will be imposed upon Part II after a rigorous verification of the accuracy of the results has been calculated.

166 C. Construction and Testing of a Delta Wing Flutter Model is being effected by replacing the actual wing by a structurally equivalent lattice network. An iterative procedure involving the evaluation of a matrix equation has been evolved for determining the bending and torsional stiffnesses of the component members of the network.

:for M.M.Chen(DIC)
:by S.Gravitz(Res.Assist. Aero.Eng.), 40 hours
:DCL Staff: Porter, 3.5 hours; WWI, 18 minutes

The routine for formulating a matrix with each element a sum of products has been completed and successfully tested.

Routines for adding matrices with each element a sum of coefficients of unknown terms and for equating two matrices term-by-term to form a set of simultaneous algebraic equations were observed to entail minor modifications of subroutines previously developed for this problem.

A routine for matrix multiplication with one matrix having elements composed of coefficients of unknown quantities has been completed and is in the process of being tested. An extension of this routine to matrices too large to fit entirely into high speed storage is being evolved.

The program for the future will consist of coordinating the subroutines previously evolved into an integrated program for the ultimate purpose of solving the wing simulation problem.

168 D. Indicial Downwash behind a Two-Dimensional Wing. In the analysis of the response of an airplane to a sharp-edge gust, and particularly in the calculation of stresses in the horizontal tail, it is important to know the downwash at the tail caused by the lift response of the wing to the gust. In the present solution for the downwash behind a two-dimensional wing, something more than the "indicial downwash" is sought. This is effected by allowing the gust front itself to have an arbitrary horizontal velocity which, combined with the velocity of the airplane, results in the wing's penetrating the gust front at a velocity V.

:for Professor H. Ashley, Aero. Eng. Dept., DIC 6727
:by N.P.Hobbs, 60 hours
:DCL Staff: Porter, 3.5 hour; WWI, 18 minutes
The function $C_1(A)$ (see Biweekly Report of March 7, 1954) has been computed on Whirlwind and found to be unity. The majority of the past two weeks has accordingly been spent in proving rigorously that $C_1(A) = 1$. Having proved this to be true, the outer integral involving $C_1(A)$ was evaluated analytically.

The remaining problem is to evaluate numerically $C_{II}(A,X^*)$ and the outer integral involving $C_{II}(A,X^*)$.

171 C. Improved Power Spectrum Estimates are to be obtained by investigating modifications to existing techniques for the numerical calculation of Fourier cosine transforms to minimize the effects of truncation and to supply a confidence curve, based upon the internal variations of the calculations, which will help to evaluate the significance of the resulting spectrum estimate. The object is to obtain a method which will give significantly better power spectrum estimates than can be obtained using existing techniques.

:for J.E. Ward, Project Engineer, Servomechanisms Laboratory
:by D.T. Ross (DIC), 100 hours; Hamilton, 20 hours
:DCL: WWI, 126 minutes

In the proposed procedure, a series of spectrum estimates, called the First Spectrum, Second Spectrum, etc., will be calculated. Each spectrum is closer to the true spectrum than the previous one by a factor which is proportional to the truncation length, $T$. The unnormalized first spectrum $S_1^*(w,T)$ is the standard estimate without any modification

$$S_1^*(w,T) = 2 \int_0^T f(t) \cos wt dt,$$

where $f(t)$ is the function to be transformed. The unnormalized $n^{th}$ Spectrum is recursively defined in terms of the $(n-1)^{st}$ Spectrum by

$$S_n^*(w,T) = \int_0^T t S_{n-1}^*(w,t) dt.$$

The normalized spectra are then defined by

$$S_n(w,T) = \frac{(2n)!}{2^n n! T^{2n-1}} S_n^*(w,T).$$

Thus it is theoretically possible, for a given truncation length $T$, to obtain a spectrum estimate for the given section of the function (from 0 to $T$), which is accurate to any specified tolerance, merely by taking $n$ large enough.

The measure of indecision which will be tested as part of this study will be the amount of oscillation in the process over and above the oscillation which is normal for the function $\cos \nu t$. Possible forms for this measure of indecision which are easy to compute at the same time as the spectra are computed are

$$\frac{1}{T} \int_0^T | \cos^2 \nu t - f(t) \cos \nu t | dt$$

and
\[
\frac{1}{T} \int_0^T (\cos^2 \nu t - f(t) \cos \nu t)^2 \, dt
\]

where, again, \( f(t) \) is the function being transformed. Both forms will be tested.

Every method for obtaining spectra by computational means can give only estimates of the true spectrum. Hence, such a process may be considered to have a certain filter characteristic, which we call the "spectral window" of the process. The spectral window of any such process when the frequency \( \nu \) is being investigated may be taken as

\[
u'(w) = \sum_0^T h_k(\nu) \cos kw \cos kw
\]

where \( k \) is the index of the digital process (corresponding to the time variable \( t \)) and \( h_k(\nu) \) includes all coefficients and special operators of the process which are used when the frequency \( \nu \) is the scanning frequency. Thus \( \nu'(w) \) measures not only the convolving function (e.g., \((\sin x)/x\)) for ordinary truncation, but also the effects of the actual numerical processes used. Note that it is obtained by inserting the function \( \cos \nu t \) into the process and hence is easily obtained. All stages of this investigation will be tested by finding \( \nu'(w) \) but final evaluations will be based upon the performance with complicated sums of cosines, which approximate autocorrelation functions actually found from nature.

For testing purposes, a program to generate functions of the form

\[
\sum_{i=1}^n A(w_i) \cos w_i t
\]

for an arbitrary number of frequencies, \( w_i \), has been written. The present version allows the use of up to fifty frequencies with arbitrary amplitudes. It will thus be possible to approximate the types of autocorrelation functions actually found from natural processes by functions with known spectra.

The Improved Fourier Transform program using weighted means appears to be operative. Several minor logical and technical errors were found and corrected. The second (equivalent) program which uses pre-multiplication by suitable weighting functions must still be tested. The program to generate sums of cosines for use as test functions has been tested and the latest version should be correct. Now several test runs will be made to determine the characteristics of the programs, the method and to establish sound procedures for general application.

172 B. **Overlap Integrals of Molecular and Crystal Physics.** Two-center overlap integrals are to be evaluated between various Slater atomic orbitals, which are of the form: (power of \( r \)) \( (\exp -ar) \times (spherical \ harmonic) \). By use of prolate spheroidal coordinates, formulas in terms of simple functions can be derived for these integrals but are unsatisfactory because they are of formidable complexity and have false singularities. The integrals are evaluated by recasting the expressions in terms of spherical Bessel functions of imaginary argument, which can be generated by a high-speed computer.
In the Tight-Binding approximation applied to a solid, two-center integrals are the most important quantities to be evaluated. Heretofore, these evaluations have been a major limitation of the method. Although in principle, closed analytic forms have been developed for these integrals, (C.C.J.Roothaan, Jour. of Chem. Phys., 19, 1445 (1951)) in practice the formulas are not only formidable but are very poor for computational purposes because of subtractions of large numbers and the resultant low accuracy. These integrals have been the object of much analytical work in the past, and if capable of being evaluated in a simple fashion, would be of great interest to physicists especially in the fields of Solid-State and Molecular Structure.

The basic subroutines of the problem have been coded and tested using CS II. The final program for the overlap integral subroutine has been written but must still be typed and given a final check. The more educational programming mistakes were:

1) Any cycle instruction with a buffer address will not operate correctly.

2) The subroutine LSR FUL for e^x, calculates e^|x| regardless of the sign of x, and then divides the result into 1 if x is negative. Consequently, for |x| \geq 50., even with x negative, the routine fails on an interpretive overflow during an its2t order. One solution of this difficulty is to modify the subroutine by replacing the 2t addresses by a buffer address.
1.3 Operating Statistics

1.31 Computer Time

The following indicates the distribution of WWI time allocated to the S&E Group.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Programs</td>
<td>70 hours, 23 minutes</td>
</tr>
<tr>
<td>Conversion</td>
<td>24 hours, 06 minutes</td>
</tr>
<tr>
<td>Magnetic Drum Test</td>
<td>25 minutes</td>
</tr>
<tr>
<td>Magnetic Tape Test</td>
<td>41 minutes</td>
</tr>
<tr>
<td>Scope Calibration</td>
<td>40 minutes</td>
</tr>
<tr>
<td>Demonstrations(#131)</td>
<td>14 minutes</td>
</tr>
</tbody>
</table>

Total Time Used           96 hours, 29 minutes
Total Time Assigned        116 hours, 30 minutes
Usable Time, Percentage   82.7%
Number of Programs         334

1.32 Program Time Distribution

The following table attempts to show how the WWI time expended on S&E programs was distributed with respect to machine runs that gave meaningful results (productive computer time) and runs that gave unsatisfactory results (lost computer time). Productive computer time is subdivided to indicate the time involved in actual computations as contrasted with the time expended in getting information out of WWI. Computer time lost is subdivided to show the portion of time lost due to errors in the programmer's formulation of his problem (logical errors); due to errors in the programmer's use of the WWI code, GS Conventions, etc. (technical errors); due to tape preparation errors; due to errors by the S&E computer operators in running the program; due to malfunctioning of terminal equipment; and finally, due to miscellaneous causes.

These times are determined as percentages of the time listed above in section 1.31 for programs. The times used in computing these figures are extracted from the biweekly report forms submitted by the various programmers who have used S&E allocated WWI time.

1. Productive Computer Time
   - Computation: 48.1%
   - Output: 12.7%

2. Computer Time Lost Due to Programmers Errors
   - Technical: 17.3%
   - Logical: 12.0%

3. Computer Time Lost Due to Other Difficulties
   - Tape Preparation: 2.0%
   - Operators' Errors: 1.4%
   - Terminal Equipment Malfunction: 4.0%
   - Miscellaneous: 2.5%

1.33 Tape Preparation

An attempt is being made to obtain some idea of the time expended in the preparation of tapes. During the past biweekly period a check was made
Due to the variations in procedures involved we have distinguished among original complete tapes and the following three types: **typed modifications** - changes of 11 or more registers which must be typed, converted and then attached to the main program or changes which must be made in the body of a Flexowriter tape; **manual modifications** - changes punched directly in 556 form and attached to a converted tape; **combined tapes** - which require duplication of two or more complete tapes.

The following information was compiled:

<table>
<thead>
<tr>
<th></th>
<th>Complete</th>
<th>Typd</th>
<th>Manuals</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Tapes</td>
<td></td>
<td>103</td>
<td>67</td>
<td>46</td>
</tr>
<tr>
<td>No. of Registers</td>
<td>21606</td>
<td></td>
<td>6048</td>
<td>340</td>
</tr>
<tr>
<td>Time Consumed</td>
<td>58hrs.13min.</td>
<td>51hrs.59min.</td>
<td>7hrs. 45min.</td>
<td>6hrs. 3min.</td>
</tr>
</tbody>
</table>

Thus, it may be seen that the average length of an original complete tape is 167.7 registers requiring 33.9 minutes to prepare. A typed modification averages 13.2 registers in length and requires 25.9 minutes to prepare which Manual Modifications average 7.4 registers and require 10.1 minutes for preparation.

In addition to the time accounted for above, a total of 12 hours and 45 minutes was spent printing computer results for photographic reproduction.

2. **ACADEMIC PROGRAM**

**CS Programming Course**

The CS introductory two-week programming course began on March 15 with an enrollment of 18 students. These students represent the following groups: Aeroelastic and Structures Research Laboratory, Instrumentation Laboratory, MIT Electrical Engineering Department, MIT Chemical Engineering Department, Dynamic Analysis and Control Laboratory, MIT Mathematics Department, Lincoln Laboratory, Geophysical Analysis Group, and the Laboratory for Nuclear Science and Engineering.

Several chapters of the Manual being used for the first week's work are being rewritten to conform with CS II conventions. It is expected that this manual will have been revised before the course is given again.

**Seminar on Advanced Programming Techniques of the Digital Computer Laboratory Staff.**

On Friday, March 12, Dr. H. H. Denman discussed the uses of magnetic tape memory and its associated delayed printer equipment.

Mr. P.R. Bagley described the Memory Test Computer on March 19.

On Friday, April 9, Professor C.W. Adams will talk on "Recent Developments in the Digital Computer Field Including Current Trends and New Computers".
S&EC Movie

The script for the movie "Making Electrons Count" has been rewritten and "shooting" will begin on the revised version of the film within a few days.

Industrial Liaison Conference

A conference on "Control Applications in Business and Industrial Systems" will be held April 5-6 at MIT under the sponsorship of MIT's Industrial Liaison Office. Those who plan to attend should contact Miss Wellington at extension 2693 immediately.

Seminar on Computing Machine Methods

On Tuesday, March 9, E. A. Robinson, director of the Geophysical Analysis Group at MIT, spoke on "Petroleum Exploration and Digital Computers.

On Tuesday, April 6, Professor James B. Reswick of the MIT Department of Mechanical Engineering will speak on "An Electronic Analog Device For Delay Line Synthesis". The seminar is held at 4 PM in Room 12-182.
3. COMPUTER ENGINEERING

3.1 WWI System Operation (L.L.Holmes, A.J.Roberts)

A prototype of the new core-memory sense amplifier is nearly complete. It is planned to install the unit in WWI for tests on Tuesday, 16 March. If the results of the tests are satisfactory, the unit will be left in service for a reliability check.

All of the fixed-voltage switching panels in Rack P0 have been replaced by a new model. This is the initial step in our plan to replace all of the fixed-voltage switching panels and voltage-variation switching panels in P Row. The models contain more reliable relays and should therefore ease our relay maintenance work load.

During the past biweekly period, the majority of the systems testing time was used to investigate the main control section of the computer in search of the origin of several transient transfer-check alarms that have occurred. Several faulty tubes and crystals were located, but the trouble is believed to be still present. More testing will be done during the systems-testing period.

3.11 Typewriters and Paper Tape (L.H.Norcott)

Commercial Controls Corp. reports that our three new Flexowriters are now going through their final assembly line and will be shipped to us shortly. As soon as they are received, the Flexo shop will modify them for use with Whirlwind.

3.2 Terminal Equipment (R.H.Gould)

One K1084P7M 16-inch cathode-ray tube that was removed from service because of spurious emission from the control grid has been successfully rehabilitated. R-F bombing of the control grid stopped the grid emission without apparent damage to the cathode.

Logical difficulties with block control have recently come to light, and a rather basic redesign of block control may be desirable. Other changes to in-out control to simplify magnetic-tape operation and to simplify in-out control generally are being planned.

Work continues on bringing in-out drawings up to date.

3.21 Buffer Drum System (K.E.McVicar)

The buffer drum is now connected to the computer at all times and is available to programmers without special arrangement.

The auxiliary-storage section of the buffer drum does not yet have a parity digit, but it is checked daily and can be used with reasonable assurance of reliable operation. The interlace on the auxiliary-storage section of the buffer drum is eight instead of four as on the auxiliary drum. It takes about twice as long to make a complete block transfer of one group from the buffer as it does from the auxiliary.
3.22 Magnetic Drums  (H.L. Ziegler)

Installation of electronic-write switching of heads in the auxiliary drum continues to progress satisfactorily. All hardware is on hand, and most of it is installed. Filament and d-c power wiring are nearly complete, and the bulk of the signal leads have been made up into cable form. These leads are to be placed in the wiring ducts during installation time Monday, 15 March, after which final wiring of the chassis plugs can begin almost immediately.

The magnetic drum-PETR monitor system is also nearing completion. As presently planned, monitoring will be available for the entire auxiliary drum, groups 4-7 of the buffer drum, and the PETR output channels. Included in the new system is an entirely new switching panel designed to consolidate the present widely scattered controls and also to provide some desirable interlocking of these controls.

3.23 Magnetic Tape  (E. P. Farnsworth)

The unit 2 delayed print-out equipment and test programs have been checked out and can be plugged into the second Flexowriter when the enclosure is completed. Switches to interchange unit 2 and unit 3 IOG, unit 2 and unit 3 print-outs, and to convert the unit 2 Flexowriter to standard FL or delayed print-out are under construction in the shop. The rewind indicator gong is installed and operating, and "ON LIMIT" lights are being mounted on each tape-unit control panel. Modifications are being made to improve the two-tone chime effect to prevent confusion with the computer alarm, and to change the "Rewind Unit 0" switch near PETR from a toggle to a push-button switch.

The delayed print-out functional schematic drawing was completed, and the block schematic for the entire magnetic-tape system has been brought up to date. Both drawings are in the WII service file.

3.24 Flexowriter Equipment  (M. Demurjian)

Discussions were held with Norcott and Carrill on the grounding of Flexowriter equipment. It was decided that all Flexowriters would have special Hubbell plugs attached and the wall receptacles would be replaced by Hubbell type wherever Flexowriters are used. For the mobile uses adapters would be available.

3.25 Magnetic Tape Equipment  (M. Demurjian)

The new Delayed Output box is in Test Control. As soon as the control boxes and cables are completed, Al Perry will connect it to the Magnetic Tape Units. The expected date of delivery for these is April 2, 1954.

Perry located and remedied the cause of spurious digits and characters which were appearing in the delayed output.
4. **ADMINISTRATION AND PERSONNEL**

**New Non-Staff** (R. A. Osborne)

Renee Feinstein is a new secretary in the Publications Office.
Lester Gediman is a new member of the Drafting Department.

**Terminated Non-Staff**

Jean Pfaff
Mildred Stickney

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<td></td>
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