Artificial Intelligence Project--RLE and M.I.T. Computation Center
Memo 20--Puzzle Solving Program in LISP
by
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In this note we give as an example of LISP programming a
function for solving a class of puzzles in a recent prize contest.

A linear graph is given with a starting point and an endpoint.
Associated with each vertex of the graph is a numerical value. The
object is to find a path on the graph from the starting point to
the endpoint which goes through no vertex twice and which maximizes
the sum of the values associated with the vertices through which
the path goes.

Example:

```
       750
      /  \
  A1-8 350  A2
   / \    /\    /\
 200 100 300 700
  A4  A5  A6  A3
   \  /  \  /  \
 550 650 550 100
  A8  A9  A7  A10
```

We shall use the following conventions in our LISP program
for solving the problem:

1. The graph is described by assigning to each vertex an
   atomic symbol and giving a list of terms one for each vertex, each
   of which is a list of three items: the name of the vertex, a list
   of the vertices immediately accessible from it, and the value
   assigned to the vertex. The graph in the example is described by:
   $$((A_1, (A_2, A_5), 7.5), (A_2, (A_1, A_5, A_9, A_3), 3.5), (A_3, (A_2, A_6), 7.0), (A_4, (A_5, A_9, A_3), 2.0)), (A_5, (A_1, A_4, A_2), 3.0), (A_6, (A_3, A_9, A_7), 6.0), (A_7, (A_6, A_9, A_{10}), 6.5), (A_8, (A_4, A_9), 5.5), (A_9, (A_8, A_4, A_2, A_6, A_7, A_{10}), 8.0), (A_{10}, (A_7, A_9, H), 1.0))$$
2. A position is described by list of the vertices covered up to that point in reverse order. The initial position is \((A_1)\) in the example on the previous page.

3. The list of squares to which it is legal to move in a position \(p\) is given by

\[
\text{legals}(p) = \text{less} \left[ \text{car} \right] \text{ass2}(\text{car}(p); \text{px1}); p
\]

where

\[
\text{less}(m; n) = \{ \text{null}(m) \to \text{NIL}; \text{occur}(\text{car}(m); n) \to \\
\text{less}(\text{cdr}(m); n); T \to \text{cons}(\text{car}(m); \text{less}(\text{cdr}(m); n)) \}
\]

and

\[
\text{ass2}(x; m) = \{ \text{eq}(\text{caar}(m); x) \to \text{cdr}(m); T \to \\
\text{ass2}(x; \text{cdr}(m)) \}
\]

and

\[
\text{occur}(x; n) = \text{null}(n) \land \{ \text{eq}(x; \text{car}(n)) \lor \text{occur}(x; \text{cdr}(n)) \}
\]

Here \(\text{less}(m; n)\) is a list of those elements of \(m\) not members of \(n\). \(\text{ass2}(x; m)\) is \(\text{cdr}\) (the first element of \(m\), \(\text{caar}\) of which is \(x\)). \(\text{occur}(x; m)\) is the predicate that asserts that \(x\) is a member of \(m\).

4. The best path is given by \(\text{best}[\text{initial position}; \text{moves from the initial position}; \text{path with very low value}]\) or in the present example, \(\text{best}[(A_1); (A_2, A_3); (A_4)]\). The function \(\text{best}[p; m; s]\) is the best path starting with the subpath \(p\), where the moves from \(p\) are to be taken from the list \(m\) and \(s\) is the best path found so far.

We have

\[
\text{best}(p; m; s) = \{ \text{null}(m) \to s; T \to \text{best}(p; \text{caar}(m); \text{null}(n); \text{eq}(\text{car}(n); \\
\text{H}) \to \text{better}(p; s) \to p; T \to s); T \to \text{best}(n; \text{legals}(n); s)\}\} \text{cons}(\text{car}(m); p))
\]

For comparing two paths, we have

\[
\text{better}(p_1; p_2) = \text{eq} \text{MINUS} \text{car} \text{SUM} \text{ADDUP}(p_2); \\
\text{LIST} \text{MINUS} \text{ADDUP}(p_1)))
\]

where

\[
\text{ADDUP}([p]) = \{ \text{null}(p) \to 0.0; T \to \text{SUM} \text{VALUE} \text{car}(p); \text{ADDUP} \text{cdr}(p))
\]

and

\[
\text{VALUE}(sq) = \text{cdr} \text{ass2}(\text{eq}; \text{px1})
\]
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