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Analysis of Static and Quasidynamic Behavior of Magnetostatically Coupled Thin Magnetic Films

Abstract: When two superposed films exert fields on each other, their static and dynamic behaviors change. The following method of analysis is used to study the behavior: The stable states are found by minimizing the total free energy of the films. Then constant-field contours are plotted in the θ_1-θ_2 plane (θ's being the stable orientations of the magnetization vectors). In examining the plot, one can predict multiple stable states, switching, threshold, hysteresis, and the detailed paths of magnetization change as a function of applied field.

The solution is carried out by a numerical process which permits evaluation of the following effects: the variation of the degrees of symmetries of the anisotropy energies, the relative orientation between the films, the coupling strength, and the drive-line layout. An example is carried out in sufficient detail for illustrative purposes.

Introduction

Stoner and Wohlfarth[1] derived the critical curve theory for a uniaxial single domain undergoing rotational flux reversal. Its application to magnetic thin films indicates several properties useful for memory and logic schemes. Subsequent investigations on Ni-Fe films reveal some fundamental deviations from the theory.* Nevertheless, the theory provides an essential framework for depicting the flux reversal behavior of thin films.

The Stoner-Wohlfarth theory has been referred to in various forms in the literature of thin magnetic films, for example the work of Bradley and Prutton.[2] However, an excellent exposition in a form which is particularly pertinent to the present work can be found in an unpublished report by Slonczewski.[3] Behringer[4] later made a significant extension to films with anisotropy energy characterized by $K_n \sin^2 n\theta (n > 1)$. Slonczewski's theory is summarized in Appendix I for reference.

As broader and deeper understanding is gained for single films, current interest has branched into multi-film structures. For instance, two films may be superposed in closely spaced parallel planes. The spacing is large enough to prevent atomic interaction but small enough to permit magnetostatic interaction. The strength of interaction depends on the thickness-to-diameter ratios and the magnetizations of the films. The magnetostatically coupled films exhibit static and dynamic behaviors significantly different from those of the individual films. In the literature, only one application[5,6] of the two-film structure has been reported; viz., an NDRO scheme using a film with high threshold for storing information and another film with low threshold for read-out.

Coherent rotational flux reversal for single films is described by critical curves and hysteresis loops which predict multiple stable states, switching threshold and hysteresis. In the two-film structures, similar information is desired. However, as the orientations of the two magnetization vectors have to be represented by two dependent variables, the mathematical formulation of the problem and the presentation of the results are very much different from those for the single-film theory.

The present paper develops a new method of analysis. For the convenience of the reader, similarities to single-film theory are indicated whenever possible. On the other hand, new methods (e.g., the constant-field contours) and new emphases (e.g., critical state being the limiting case of stable state) are stressed. The mathematical formulation accommodates the many degrees of freedom provided by the two-film structure; namely, the degrees of symmetries of the anisotropy energies, the relative orientation between the films,

* See comprehensive review by S. Middelhoek, p. 394, this issue.

the coupling strength and the drive-line layout. The assumptions and the limitations implied are also discussed in some detail.

Assumptions

In order to facilitate analysis, the following assumptions are made:

1. Each film behaves as a single domain with uniform magnetization and can therefore be represented by a single magnetization vector.
2. Flux change is effected by the rotation of the magnetization vector. This can be realized under either of the two following situations: The threshold field for wall motion is much higher than that required for rotation; or the wall motion is much slower than the rotation, such that for an applied pulse of short duration only the rotation can take place.
3. The magnetization vector of each film is confined to the plane of the film; otherwise an excessive magnetization energy would result from a magnetization component perpendicular to the film plane.
4. The magnetization of each film produces an internal demagnetizing field. Moreover, the two superposed films are in such proximity that each is exerting a planar uniform field throughout the other. Based on formulae given in Bozorth[7] and Chang[8] both the internal and the external fields for a thin film are

$$\mathbf{H} \simeq -(\pi/4)(t/d)\mathbf{M}\,, \tag{1}$$

where $\mathbf{H}$ = uniform planar magnetic field inside or outside of the film due to its magnetization
t = film thickness
d = film diameter
$\mathbf{M}$ = uniform magnetization of the film.

Calculations indicate that the above equation holds in the space roughly covering the film area and of a depth of one-tenth the film diameter.

The internal field, as given by Bozorth, was confirmed experimentally by Humphrey,[9] and the expression for the external field was later confirmed experimentally by Matick.[10]

5. The anisotropy energy of each film is assumed to be of the form $K_n \sin^2 n\theta$, where K_n is the anisotropy constant, n the number of axes of symmetry, and θ the angle between the magnetization vector and the major axis of symmetry.

Physical principle and mathematical model

The physical structure under consideration is two superposed films with their major easy axes at an angle α (see Fig. 1). A stripline wraps both films, or just one film, to provide the same or different fields to the films. More than one stripline are used to provide fields in different directions or to sense various components of flux changes.

The purpose of the present analysis is to find stable orientations of the films for a given applied field as well as the threshold for rotational switching. Other

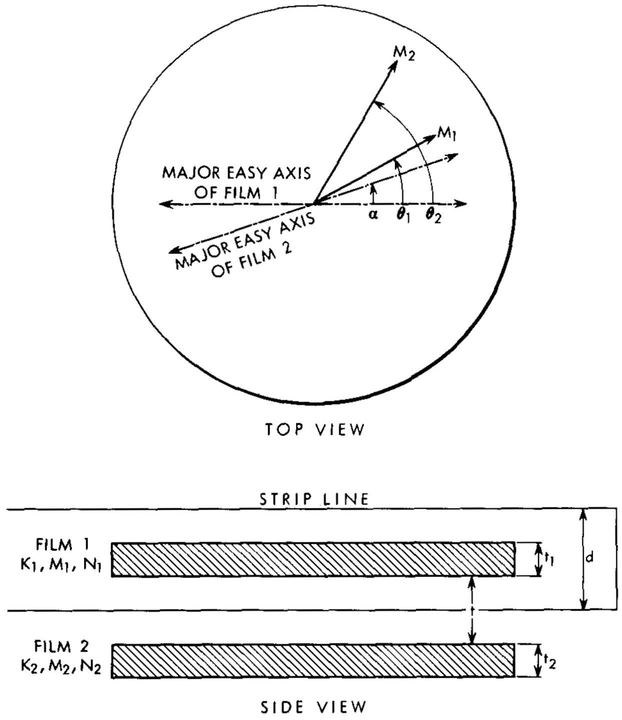


Figure 1 **Two superposed films.**

useful relations are derivable from the above information.

The physical principle utilized for the analysis is that a system tends toward minimum energy for its stable state. The total free energy of the two-film structure consists of the individual anisotropy energies, the individual magnetization energies, and the interaction magnetization energy. In normalized form, as derived in Appendix II, the total free energy is:

$$\begin{aligned} e = &-[\cos\theta_1 + sm_2\cos\theta_2]h_x \\ &-[\sin\theta_1 + sm_2\sin\theta_2]h_y \\ &+h_i\cos(\theta_1-\theta_2) \\ &+\sin^2 p\theta_1 \\ &+k_{2q}\sin^2 q(\theta_2-\alpha)\,, \end{aligned} \tag{2}$$

where

e = the total free energy of the coupled films
(θ_1, θ_2) = the orientations of the magnetization vectors
(h_x, h_y) = the x-, y-components of the applied field
h_i = the interaction field
α = the angle between the easy axes of the films
m_2, k_{2q} = the ratios of magnetization and anisotropy constants of the films
p, q = the degrees of symmetry in the anisotropy energies of the films
s = a constant determined by the layout of the drive lines.

Table 1 **Conditions for equilibrium**

	Conditions at (θ_1, θ_2)		*Then* $e(\theta_1, \theta_2)$ *is*	*Physical significance*
$e(\theta_1, \theta_2)\varepsilon c^2$	$\frac{\partial^2 e}{\partial\theta_1\partial\theta_2} - \frac{\partial^2 e}{\partial\theta_1^2}\frac{\partial^2 e}{\partial\theta^2_2} < 0$	$\frac{\partial^2 e}{\partial\theta_1^2}\left(\text{or } \frac{\partial^2 e}{\partial\theta_2^2}\right) < 0$	Relative maximum	Unstable
		$\frac{\partial^2 e}{\partial\theta_1^2}\left(\text{or } \frac{\partial e^2}{\partial\theta_2^2}\right) > 0$	Relative minimum	Stable
$\frac{\partial e}{\partial\theta_1} = \frac{\partial e}{\partial\theta_2} = 0$	$\frac{\partial^2 e}{\partial\theta_1\partial\theta_2} - \frac{\partial^2 e}{\partial\theta_1^2}\frac{\partial^2 e}{\partial\theta_2^2} > 0$		Saddle point	Unstable

The normalization factors and explicit definitions of the above quantities are given in Appendix II.

The mathematical apparatus required to find the relative minimum energies is shown in the calculus theorems[11] in Table 1.

As indicated in the table, $e(\theta_1, \theta_2)\varepsilon c^2$, the function $e(\theta_1, \theta_2)$ has continuous first and second deviatives.

Stable states

According to Table 1, the extrema are the solutions of

$$\frac{\partial e}{\partial\theta_1} = \frac{\partial e}{\partial\theta_2} = 0\,, \tag{3}$$

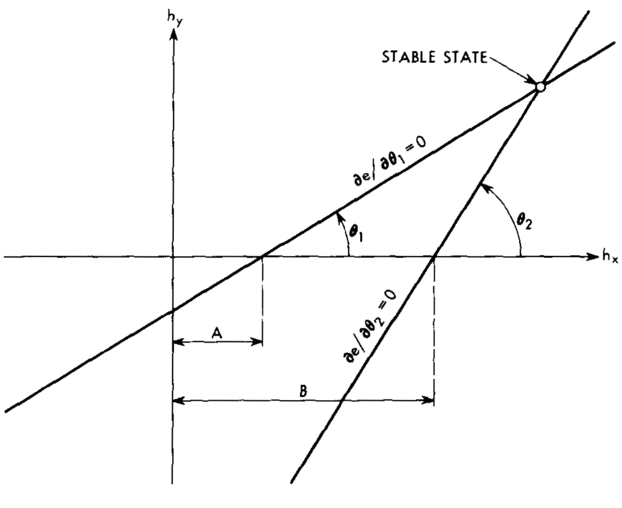
which lead to two equations relating (h_x, h_y) and (θ_1, θ_2). With (h_x, h_y) given, (θ_1, θ_2) can be found; conversely, with (θ_1, θ_2) given, (h_x, h_y) can be found. However, since the equations are linear in h_x and h_y, it is easier to solve for (h_x, h_y) for given (θ_1, θ_2) either by graphical construction (Fig. 2) or by determinants. The solution obtained may be either a stable state (relative minimum) or an unstable state (relative maximum or saddle point) depending on the values of $\Delta = e_{12}{}^2 - e_{11}e_{22}$ and e_{11} (see Table 1).

In Fig. 2, it is interesting to note that the two lines representing $e_1 = 0$ and $e_2 = 0$ have inclinations θ_1, θ_2 which are the orientations of the magnetization vectors $\mathbf{M}_1$, $\mathbf{M}_2$, respectively. It is also obvious that for given (θ_1, θ_2), (h_x, h_y) can be uniquely determined since two lines can have only one intersection. However, several pairs of lines corresponding to different solutions for (θ_1, θ_2) (multistable states) may intersect at the same point in the h_x-h_y plane. For $\theta_1 = \theta_2$, the two lines will intersect at infinity, implying that the two magnetization vectors will both be pulled to the direction of a high field.

Figure 2 **Stable state obtained by graphically solving** $\frac{\partial e}{\partial\theta_1} = \frac{\partial e}{\partial\theta_2} = 0$.

A: $\frac{1}{\sin\theta_1}[h_i \sin(\theta_1 - \theta_2) - p\sin 2p\,\theta_1]$

B: $\frac{-1}{sm_2 \sin\theta_2}[h_i \sin(\theta_1 - \theta_2) + qk_{2q}\sin 2q(\theta_2 - \alpha)]$



Critical states

For a given pair of films subject to a given field, the total free energy of the two films can be plotted as constant $e(\theta_1, \theta_2)$ contours in the θ_1-θ_2 plane. Relative minima and maxima are separated by energy barriers (see Fig. 3). The film pair is in a stable state corresponding to one of the minima. As the field is changed, the contours will alter and the stable state will shift continuously. However, when the field varies to a value to lower the energy barrier such that the minimum state will vanish, the stable state will suddenly switch into a neighboring minimum point. Such a field value is a critical one. The loci of critical field values in the h_x-h_y coordinates constitute a critical curve or rotational switching threshold curve.

In the theory for a single film, the critical state is mathematically identified as a point of inflection as well as an extremum point, and can be found by solving $\partial E/\partial\theta = 0$ and $\partial^2 E/\partial\theta^2 = 0$ simultaneously.

In coupled-film theory, such simple mathematical identification does not exist. However, since the critical

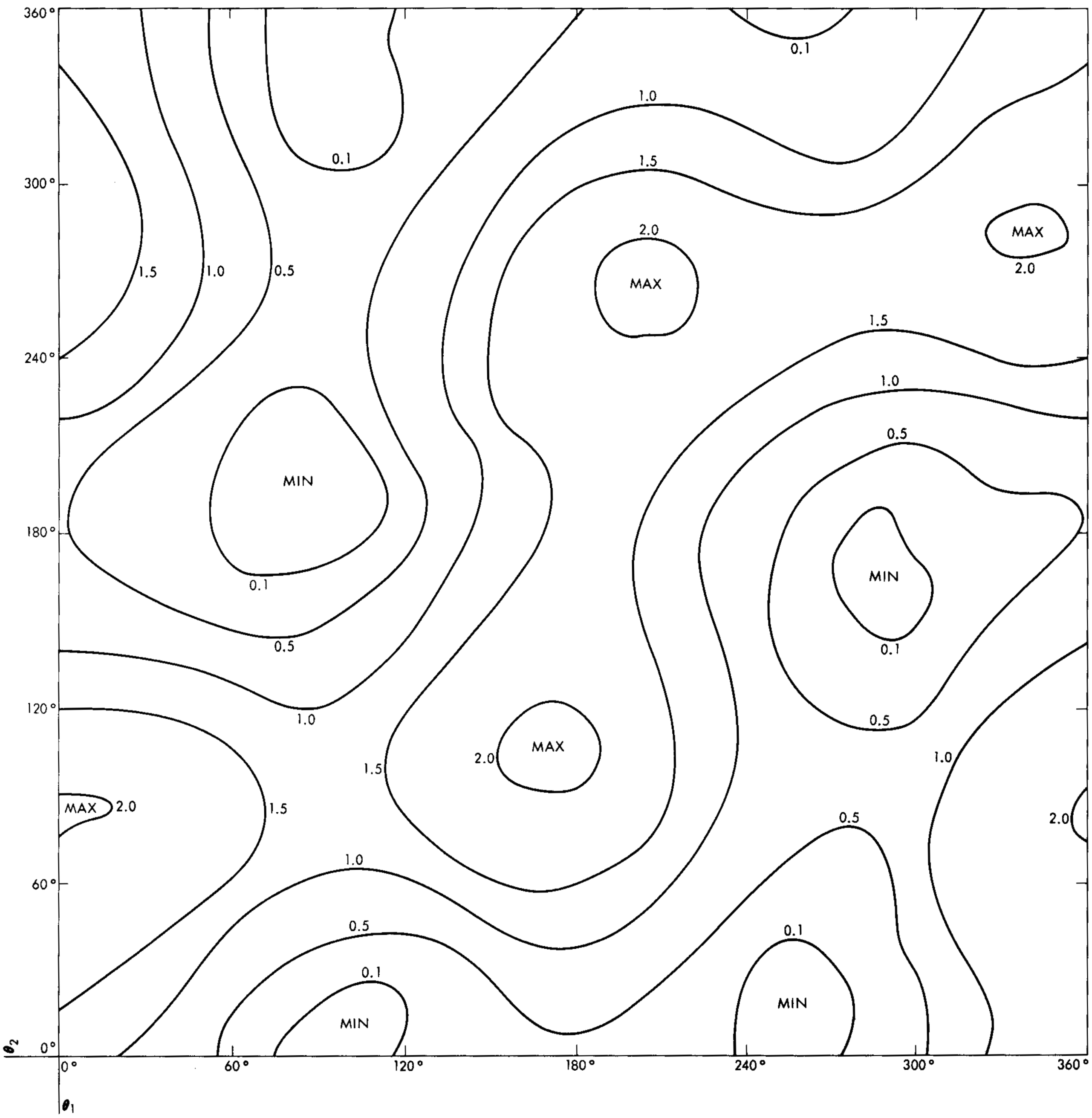


Figure 3 **Total free energy contours of a pair of magnetostatically coupled films.**

state is the limiting case of stable states, one can propose that if the stable state region is mapped out in the θ_1-θ_2 coordinates the critical state locus should be the boundary of the stable state region.

Comparison with single-film theory

Table 2 summarizes the similarities and differences between a single film and a pair of magnetostatically coupled films. One important difference not revealed in the mathematical formalism is the fact that the simple critical curve theory can not be extended to the coupled films. Consequently, new methods of analysis and presentation of results (see Section on constant-field contours in the θ_1-θ_2 plane) are different conceptual definitions and practical calculations (critical state) are called for.

Appendix I discusses the critical-curve theory for single films. It is proved that the stable states can be represented by families of straight lines in the h_x-h_y plane. The orientation of each line is the stable magnetization vector orientation, and the points on the line indicate all possible field values to maintain the magnetization orientation. The envelope of the family of equilibrium lines is the well-known astroid-shaped

Table 2 **Comparison between single film and pair of magnetostatically coupled films.**

	Single film	*Coupled films*
Magnetization vectors	$\mathbf{M} = M \angle \theta$	$\mathbf{M}_1 = M_1 \angle \theta_1$; $\mathbf{M}_2 = M_2 \angle \theta_2$
Major easy axes	$\theta = 0$	$\theta_1 = 0$, $\theta_2 = \alpha$
Total free energy	Anisotropy energy + magnetization energy	Anisotropy energies + magnetization energies (due to applied field) + magnetization energy (due to interacting field)
Stable states (corresponding to minimum energy) are determined by solving equations shown	$\partial E/\partial\theta = 0$, subject to the condition $\dfrac{\partial^2 E}{\partial\theta^2} > 0$	$\partial E/\partial\theta_1 = \partial E/\partial\theta_2 = 0$ subject to the conditions $\dfrac{\partial^2 E}{\partial\theta_1{}^2} > 0$ and $\left(\dfrac{\partial^2 E}{\partial\theta_1\partial\theta_2}\right)^2 - \dfrac{\partial^2 E}{\partial\theta_1{}^2}\dfrac{\partial^2 E}{\partial\theta_2{}^2} < 0$
Critical states	Limiting case of stable states $\partial E/\partial\theta = 0$ $\partial^2 E/\partial\theta^2 = 0$	Limiting case of stable states $\partial E/\partial\theta_1 = \partial E/\partial\theta_2 = 0$ $\partial^2 E/\partial\theta_1{}^2 = 0$ or $\partial^2 E/\partial\theta_2{}^2 = 0$ or $\left(\dfrac{\partial^2 E}{\partial\theta_1\partial\theta_2}\right)^2 - \dfrac{\partial^2 E}{\partial\theta_1{}^2}\dfrac{\partial^2 E}{\partial\theta_2{}^2} = 0$
Convenient description	Critical curve and equilibrium line in h_x-h_y plane (see Appendix I and Fig. 8)	Constant field contours in θ_1-θ_2 plane (see Section of that title and also Fig. 4)

critical curve. All equilibrium lines are tangent to and terminate on the critical curve. These geometrical relationships make the critical curve a uniquely convenient and concise method of presenting both stable states and critical states in the same Figure.

In coupled films, as is discussed in the Section on stable states (also see Fig. 2), the stable state point (θ_1, θ_2) is determined by two intersecting straight equilibrium lines in the h_x-h_y plane. There are no longer families of straight lines as stable states, with their envelope as the critical curve.

A rotational threshold curve (or critical curve) constructed in the h_x-h_y plane based on the definition in the previous Section does not relate to the stable states in any sense other than being their limiting values. A different single graphical presentation of both stable states and critical states is therefore derived in the following Section.

Constant-field contours in θ_1-θ_2 plane—prediction of switching behavior

For a given pair of films (as specified in Appendix II by m_2, k_{2q}, h_i, p, q, α, s), and a given applied field (as represented by h_x, h_y), one can find one or more stable states (as represented by θ_1, θ_2). One way of presenting all possible stable states corresponding to all possible values of fields is to plot constant-field contours in the θ_1-θ_2 plane.

Figure 4 gives an illustrative example. It considers two identical films with easy axes at 90°. Each film alone has a rotational threshold field $H_K = 2K/\mu_0 M$. We choose the thickness such that the demagnetizing field H_d is equal to 0.25 H_K. (Typically, $H_K = 4$ oe, $H_d = 1$ oe for 0.5 cm diameter, 5000 A thick Permalloy film). This results in an interacting field $h_i = 2H_d/H_K = 0.5$ (see Eq. A2.8).

The plot is to be examined to reveal such properties as multiple stable states, switching, threshold and hysteresis.

The limiting cases are to be checked first as illustrated in Fig. 5. For example, for infinite applied field at 45°, 135°, 225° or 315°, both magnetization vectors align exactly along the applied field. At a given large field, there is only one corresponding stable state. With no field applied, there are four possible quiescent stable states, Q_1, Q_2, Q_3 and Q_4. For every

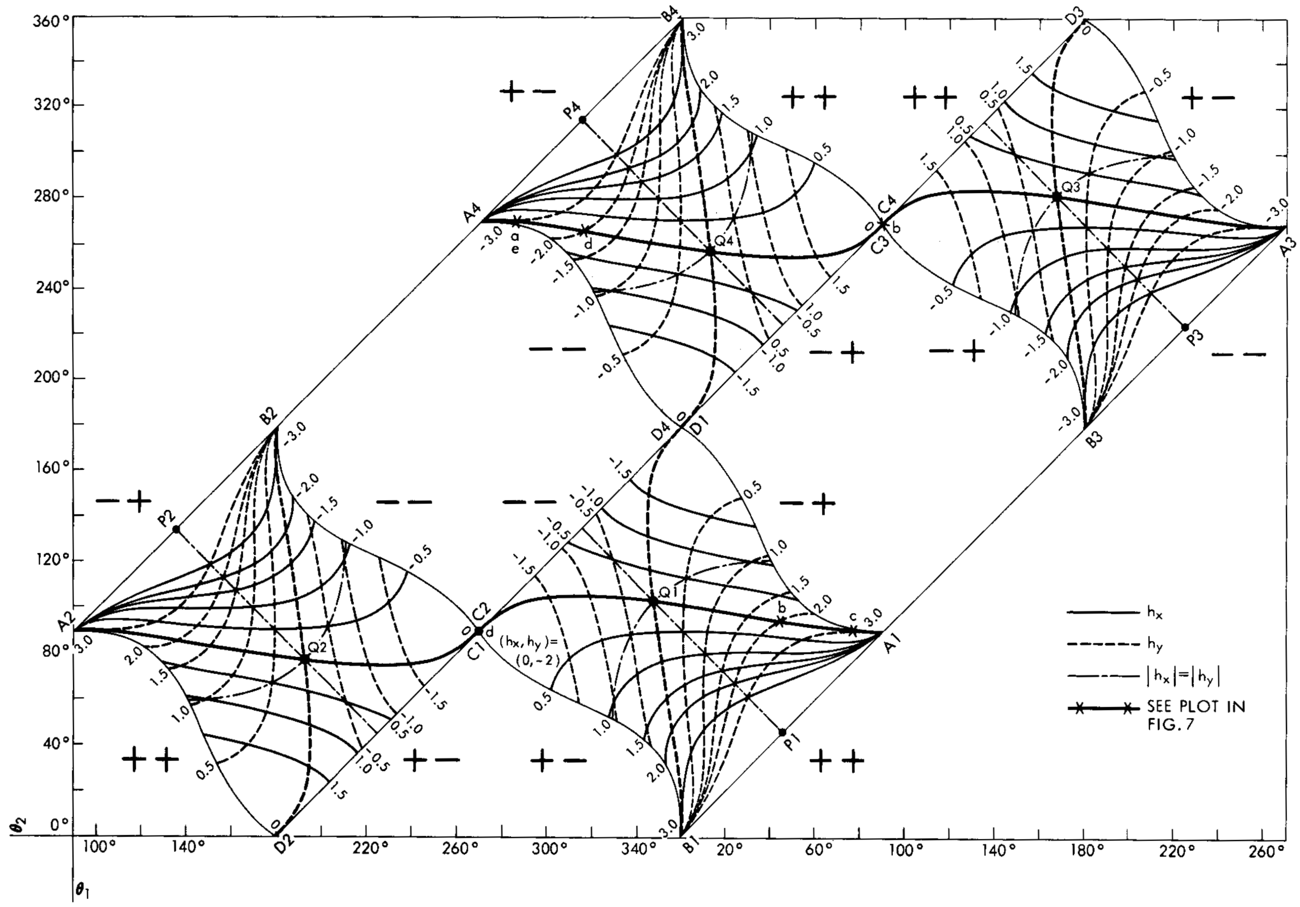


Figure 4 **Constant-field contours in θ_1-θ_2 plane.**
Field in units of $K/\mu_0 M$.

Parameters: $m_2 = 1$ $h_i = 0.5$ $q = 1$ $s = 1$
$k_{2q} = 1$ $p = 1$ $\alpha = \pi/2$

Figure 5 **Stable states at infinite or zero applied field:**

	h_x	h_y	θ_1	θ_2
P_1	∞	∞	45°	45°
P_2	−∞	∞	135°	135°
P_3	−∞	−∞	225°	225°
P_4	∞	−∞	315°	315°
Q_1	0	0	347°	103°
Q_2	0	0	193°	77°
Q_3	0	0	167°	283°
Q_4	0	0	13°	257°

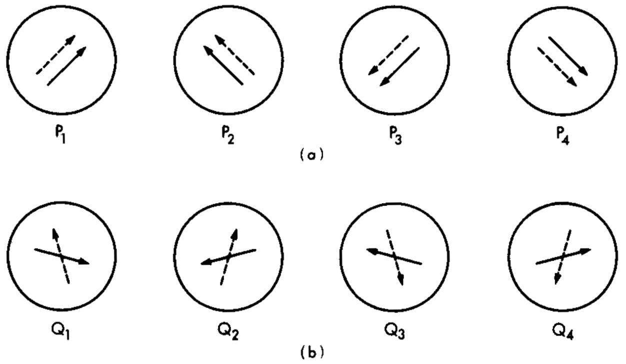
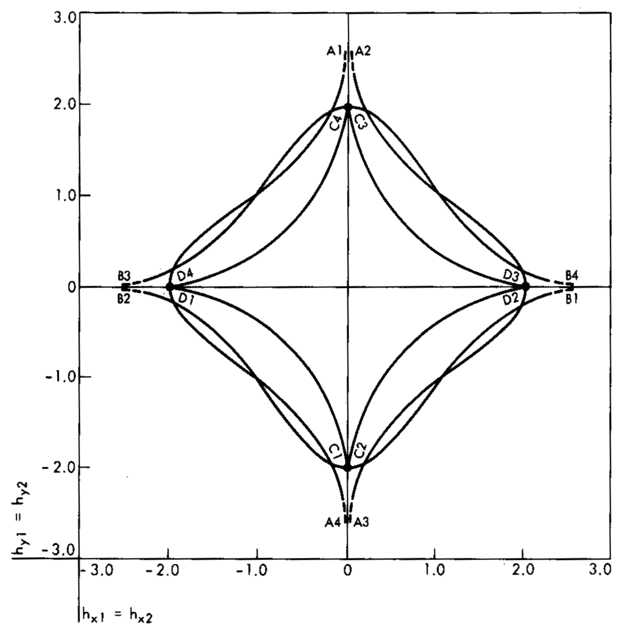


Figure 6 **Rotational switching threshold curve.**
Field in units of $K/\mu_0 M$. Parameters are same as for Figure 4.



quiescent stable state, the two magnetization vectors are nearly opposing each other while each is aligned nearly along its own easy axis. Opposing magnetization vectors minimize the demagnetizing energy and the alignment with the easy axis minimizes the anisotropy energy.

The four regions with duplicate field values clearly indicate *multiple stable states* at low fields. Now let us confine our attention to Region 1. When a field is applied, the stable state moves from the quiescent point Q_1 toward the boundary. If the total applied field is increasing in any direction between 0° and 90°, both magnetization vectors will eventually align with the applied field. Otherwise, when the stable state moves onto the boundary, it will *switch* out of Region 1. The corresponding field (h_x, h_y) is the *threshold* for rotational switching. If the switching is irreversible, there will be *hysteresis*. All possible means of switching out of Region 1 are summarized in Table 3. The boundary field values are used to plot the rotational switching threshold curve as shown in Fig. 6.

The information contained in the constant-field contours can also be used to construct *M-H* loops; one example is given in Fig. 7. The magnetic field is applied in the *y*-direction with h_x held at zero. As the field alternates in directions, the magnetization vectors switch between Regions 1 and 4. The change of (θ_1, θ_2) as a function of (h_x, h_y) is indicated by the traces *a, b, c, d, e* in the constant-field contours in Fig. 4. The *M-H* loops in Fig. 7 are labelled correspondingly.

Summary and discussion

1. The static and quasidynamic behavior of coupled films have been analyzed by plotting stable states as constant-field contours in the θ_1-θ_2 plane. The analysis

Table 3 **Possible modes of switching from quiescent stable state Q1** *(Refer to Fig. 4).*

Drive	*Leaving boundary*	*Entering region*	*Remarks*
$(+, +) \rightarrow (+\infty, +\infty)$			Approaching *A*1*B*1. No switching.
$(+, -)$ Exceeding thresholds of both *B*1*C*1 and *D*3*A*3	*B*1*C*1	4	Irreversible switching. Hysteresis.
$(0, -)$ Exceeding $(0, -2)$	*C*1 or *C*2	$4(h_x = 0^+)$ or $3(h_x = 0^-)$	Irreversible switching. Hysteresis.
$(0, 0) \rightarrow (0, -2) \rightarrow (0, 0)$		1 or 2	Equally likely to fall into Region 1 or 2. Reversible. No hysteresis.
$(-, -)$ Below threshold of *B*2*C*2 (or *D*4*A*4)	*C*1*D*1	$2(\lvert h_y\rvert > \lvert h_x\rvert)$ or $4(\lvert h_y\rvert < \lvert h_x\rvert)$	Irreversible switching. Hysteresis.
$(-, -)$ Exceeding thresholds of both *B*2*C*2 and *D*4*A*4	*C*1*D*1	3	Irreversible switching. Hysteresis.
$(-, 0)$ Exceeding $(-2, 0)$	*D*1 or *D*4	$2(h_y = 0^+)$ or $3(h_y = 0^-)$	Irreversible switching. Hysteresis.
$(0, 0) \rightarrow (-2, 0) \rightarrow (0, 0)$		1 or 4	Equally likely to fall into Region 1 or 4. Reversible. No hysteresis.
$(-, +)$ Exceeding thresholds of both *D*1*A*1 and *B*3*C*3	*D*1*A*1	2	Irreversible switching. Hysteresis.

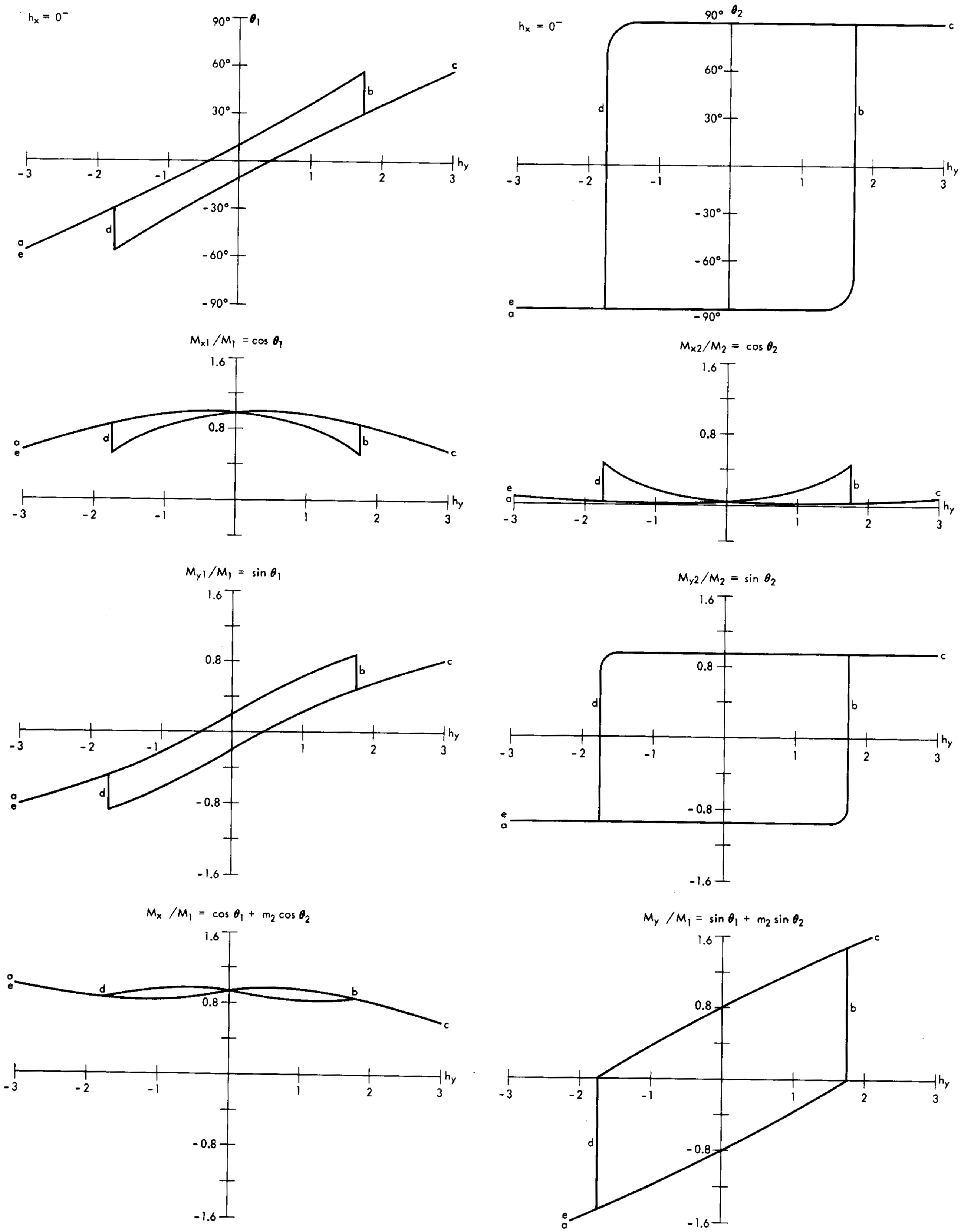


Figure 7 **M-H loops.**
Field in units of KM. The curves are based on data from Figure 4. Note the corresponding points a to e in both Figures.

can be extended to the following cases: more drive lines, each with different $s = H_2/H_1$, more than two coupling films, and films each with mixed anisotropies. These cases are all conceivably of practical interest.

2. The analysis reveals properties such as multiple stable states, switching, threshold and hysteresis for the coupled films. These suggest possible applications for memory and logic. There are three distinctive features of the coupled films: (a) The films form a closed magnetic path which allows the use of thick films with high demagnetizing field when not coupled. (b) Each film acts as a rotatable bias on the other. This may allow novel memory and logic operations. (c) The two films constitute a system of two elements with coupling which is a sine function of the angular displacement between the magnetizations.

3. In the present analysis, damping is not taken into account, hence the analysis is not truly dynamic in nature. In a separate paper[12] switching behavior with damping is studied.

4. A comprehensive study was made and will be reported elsewhere of the coupled-film behavior for various values of m_2, k_{2q}, h_i, p, q, α, and s corresponding to variations in structure, film thickness, and material property.

5. The rotational mechanism of flux reversal occurs only in limited ranges of magnitudes and angles of the applied field. A complete description of switching behavior must take into account other mechanisms. One such consideration on single, uniaxial Permalloy films is presented in Refs. 13 and 14.

Appendix I. Critical curve theory for single films[15]

The total free energy of a uniaxial film is given by

$$E = -\mu_0 \mathbf{M}\cdot\mathbf{H} + K \sin^2 \theta\,, \qquad \text{(A1.1)}$$

where

θ = orientation of $\mathbf{M}$ as measured from the easy axis
E = total free energy per unit volume
$-\mu_0\mathbf{M}\cdot\mathbf{H}$ = mutual magnetization energy per unit volume between the film and the applied field
K = anisotropy constant.

Physically, the stable orientation of a magnetization vector at a given field (H_x, H_y) corresponds to a position with relative minimum-energy. As the field changes, the minimum-energy orientation, or stable state, will also change. At certain field values, the stable state has to jump to a new value, instead of varying continuously, to achieve a new minimum-energy orientation. The locus of such field values in the H_x-H_y plane is a critical curve. As we shall presently explain, the critical curve can also be used to find stable states at any given field.

The mathematical conditions at which the stable states or the critical states occur are summarized here:

Conditions at θ		*Then $E(\theta)$ is*	*Physical significance*
$E(\theta)\varepsilon c^2$	$\partial^2 E/\partial\theta^2 < 0$	Relative maximum	Unstable
$\partial E/\partial\theta = 0$	$\partial^2 E/\partial\theta^2 > 0$	Relative minimum	Stable
	$\partial^2 E/\partial\theta^2 = 0$	Point of inflection	Switching

The equation $\partial E/\partial\theta = 0$ gives a family of straight lines in the H_x-H_y coordinates with parameter θ:

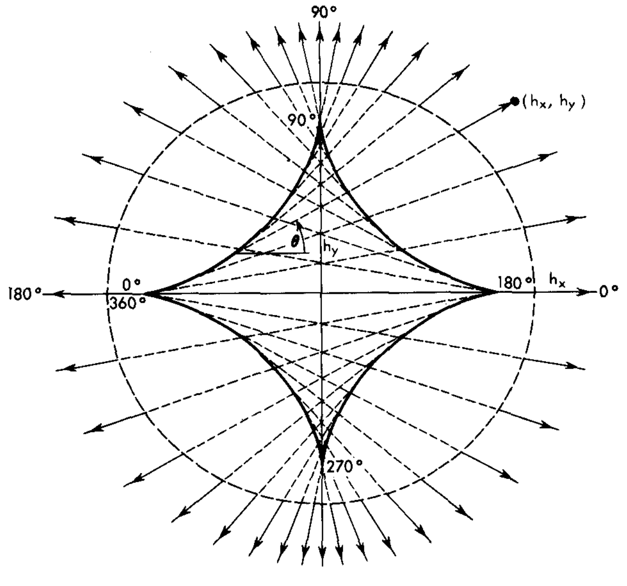
$$\mu_0 M \sin\theta H_x - \mu_0 M \cos\theta H_y + 2K \sin\theta\cos\theta = 0\,. \qquad \text{(A1.2)}$$

The simultaneous solution of $\partial E/\partial\theta = 0$ and $\partial^2 E/\partial\theta^2 = 0$ gives the envelope to the family of straight lines. By definition, these straight lines are tangent to the envelope. Further examining Eq. (A1.2), we see that the slope of the straight line is

$$\partial H_y/\partial H_x = \tan\theta\,, \qquad \text{(A1.3)}$$

where θ is the orientation of the magnetization vector. Hence the magnetization vector is parallel to the tangent to the critical curve. Since the envelope divides its tangent $\partial E/\partial\theta = 0$ into two segments, each with $\partial^2 E/\partial\theta^2 > 0$, or $\partial^2 E/\partial\theta^2 < 0$, the former segment determines the orientation of the magnetization vector. The readers are reminded that the results obtained

Figure 8 **Critical curve and stable-state lines for an uniaxial film.**
Field in units of $2K/\mu_0 M$.



above apply to films with any anisotropy energy of the form $E_k(\theta)$

If a critical curve is properly labelled (as in Fig. 8) to help select the segment of tangent with $\partial^2 E/\partial\theta^2 > 0$, the stable states corresponding to a given field can be easily determined according to the following rule: Draw a tangent from the field point (h_x, h_y) to the critical curve such that the tangent is pointing toward the field point, the orientation of the tangent is a stable state for the given field if and only if this angle, as measured from the $+h_x$ axis, falls within the range labelled on the corresponding part of the critical curve. Based on the above rule, it is found that for field values within the critical curve, one of two possible stable states can occur, depending on the history of magnetization. Outside the critical curve, only one stable state is possible. When the critical curve is crossed during field change, switching (or discontinuous change in magnetization orientation) may or may not occur. A concrete illustration of the application of the above rule is shown in Fig. 9. The easy-direction hysteresis is constructed by using field values along the h_x-axis and the corresponding $M \cos\theta$, with θ determined by the rule. The other loops are similarly constructed.

Appendix II. The total free energy of coupled films

For the isolated Film 1, the free-energy equation is the following:

$$E_1 = E_{M1} + E_{K1}$$
$$= (-\mu_0 \mathbf{M}_1 \cdot \mathbf{H}_1 + K_{1p} \sin^2 p\theta_1)\pi d^2 t/4\,, \qquad \text{(A2.1)}$$

where

E_1 = total free energy (joule)
E_{M1} = mutual energy between the film and the applied field
E_{K1} = anisotropy energy
μ_0 = permeability of air (henry/m)
$\mathbf{M}_1$ = magnetization of Film 1 (ampere-turn/m)
$\mathbf{H}_1$ = applied field on Film 1 (ampere-turn/m)
K_{1p} = anisotropy constant (joule/m^3)
p = number of axes of symmetry for anisotropy energy
θ_1 = angle between magnetization and major easy axis (radians)
d = diameter of the Films 1 and 2 (m)
t_1 = thickness of Film 1 (m).

Similarly for the isolated Film 2,

$$E_2 = E_{M2} + E_{K2}$$
$$= [-\mu_0 \mathbf{M}_2 \cdot \mathbf{H}_2 + K_{2q} \sin^2 q(\theta_2 - \alpha)]\pi d^2 t_2/4\,, \qquad \text{(A2.2)}$$

where

α = angle between major axes of Films 1 and 2
t_2 = thickness of Film 2.

Since the two films are placed in proximity and subject to the fields of each other, there is mutual magnetization energy E_{M12} between them.

$$E_{M12} = (-\mu_0 \mathbf{M}_1 \cdot \mathbf{H}_{12})\pi d^2 t_1/4$$
$$= -\mu_0 \mathbf{M}_1 \cdot (-N_2 \mathbf{M}_2)\pi d^2 t_1/4$$
$$= \mu_0[(\pi^2/16) d t_1 t_2]\mathbf{M}_1 \cdot \mathbf{M}_2\,, \qquad \text{(A2.3)}$$

where

$$N_2 = \frac{\pi}{4}\frac{t_2}{d}\,. \qquad \text{(A2.4)}$$

To further simplify the analysis, assume that the drive striplines (Fig. 1) are both between or around the films such that

$$s = \mathbf{H}_2/\mathbf{H}_1\,. \qquad \text{(A2.5)}$$

Table A.II gives the value of s for some typical configurations.

Table A.II **Drive line layout and values of *s*.**

Configuration	*Value of s*
Stripline wraps both films	1
Stripline goes between the films with return around but away from Film 1	−1
Stripline goes between the films with return around and close to Film 1	−1 to 0

The total energy of the system of the two films is

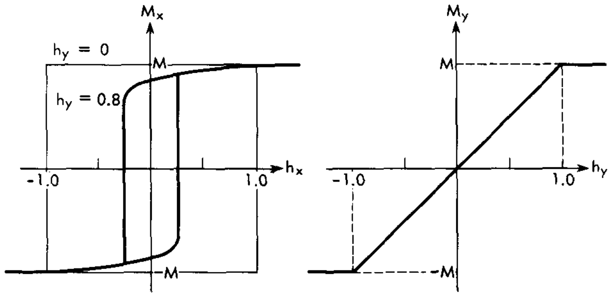
$$E_T = E_1 + E_2 + E_{M12}\,. \qquad \text{(A2.6)}$$

Substituting Eqs. (A2.1) through (A2.3) into Eq. (A2.6) and using $K_{1p}/\mu_0 M_1$, M_1, K_{1p} as normalization factors for field, magnetization, and energy per unit volume respectively, we obtain

$$e = -[\cos\theta_1 + s m_2 \cos\theta_2]h_x$$
$$- [\sin\theta_1 + s m_2 \sin\theta_2]h_y + h_i \cos(\theta_1 - \theta_2)$$
$$+ \sin^2 p\theta_1 + k_{2q} \sin^2 q(\theta_2 - \alpha)\,. \qquad \text{(A2.7)}$$

Figure 9 **Hysteresis loops.** *Field in units of $2K/\mu_0 M$.*

(a) Easy direction; (b) hard direction.



In Eq. (A2.7) we have

$$e = \frac{E_T/(\pi d^2 t_1/4)}{(\mu_0 M_1)(K_{1p}/\mu_0 M_1)}$$

$$h_i = N_2 M_2/(K_{1p}/\mu_0 M_1) = 2H_d/H_k \qquad \text{(A2.8)}$$

$$= \text{interacting field between the two films}$$

$$m_2 = (M_2/M_1)(t_2/t_1)$$

$$k_{2q} = (K_{2q}/K_{1p})(t_2/t_1)$$

$$h_x = H_x/(K_{1p}/\mu_0 M_1)$$

$$h_y = H_y/(K_{1p}/\mu_0 M_1)\,.$$

It is of interest to note that since N_2M_2 is the demagnetizing field (H_d) in Film 2, and $2K_{1p}/\mu_0 M_1$ is the rotational threshold (H_k) for an uniaxial Film 1, the interacting field h_i can also be written as $2H_d/H_k$.

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