

DESIGNER'S NOTEBOOK



Extending Fixed-Point Dynamic Ranges

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Design Problem

How can you extend the fixed-point math dynamic range beyond the range of a Q15 number with a minimum of instructions?

Solution

In many advanced control problems such as state estimators, Kalman filters and some high Q filters, the dynamic range/accuracy of the coefficient can sometimes be beyond the range of a Q15 number while the data value can be typically represented as a Q15 number.

Aside from trying to dynamically scale the coefficients to extract as much accuracy as possible or trying to use floating point math, there is a technique that can perform 32-bit \times 16-bit math at an effective 4 cycles per Tap and potentially 2 cycles per Tap for larger than 6th order systems (+ some fixed overhead of about 8-13 cycles).

The trick is to re-scale the numbers and represent the problem as an integer value + a fractional value. For example:

$$Y = 2.391456 * X_0 - 0.0235045 * X_1 + 0.000329758 * X_2 - 34.3392345 * X_3$$

In the above equation, the filter Coefficients have a dynamic range exceeding a 16-bit Q15 number. If we re-scale the problem as follows:

$$Y = [1224.425472 * X_0 - 12.034304 * X_1 + 0.168836096 * X_2 - 17581.68806 * X_3] / 512$$

And then allocate the following coefficient values:

$$Y = [(A_{0i} + A_{0f}) * X_0 + (A_{1i} + A_{1f}) * X_1 + (A_{2i} + A_{2f}) * X_2 + (A_{3i} + A_{3f}) * X_3] / 512$$

where:

$$A_{0i} = 1224 = 04C8h$$

$$A_{0f} = 0.425472 = 3676h (= 0.425476074)$$

$$A_{1i} = -12 = FFF4h$$

$$A_{1f} = -0.034304 = FB9Ch (= -0.034301758)$$

$$A_{2i} = 0 = 0000h$$

$$A_{2f} = 0.168836096 = 159Ch (= 0.168823242)$$

$$A_{3i} = -17581 = BB53h$$

$$A_{3f} = -0.68806 = A7EEh (= -0.688049316)$$

The problem then reduces to calculating the following:

$$Y = (A0i*X0 + A1i*X1 + A2i*X2 + A3i*X3) + (A0f*X0 + A1f*X1 + A2f*X2 + A3f*X3)$$

This is like calculating two filter banks. The above problem is coded in the example below:

```
; Assume:      X0,X1,X2,X3 = Q15 (-1 range 0.999053955)
;              Y = Q10 (-32 range +31.99902344)
;  Ymin-max = 2.391456 + 0.0235045 + 0.000329758 + 34.3392345
;              = +/- 36.75452476
;              Sat      = 06000h
;              Round    = 08000h

      SETC      OVM      ; Enable saturation.
      SETC      SXM      ; Enable sign extension.
      SPM       3        ; Set shift mode = -6
      LT        A0f
      MPY       X0        ; P = A0f*X0
      LTP       A1f      ; ACC = A0f*X0
      MPY       X1        ; P = A1f*X1
      LTA       A2f      ; ACC = ACC + A1f*X1
      MPY       X2        ; P = A2f*X2
      LTA       A3f      ; ACC = ACC + A2f*X2
      MPY       X3        ; P = A3f*X3
      LTA       A0i      ; ACC = ACC + A3f*X3
      SPM       0
      SACH      Temp,6    ; On C5X replace by BSAR 9
      LAC       Temp,1    ; ACC = ACC/512
      ; instruction.
      MPY       X0        ; P = A0i*X0
      LTA       A1i      ; ACC = ACC + A0i*X0
      MPY       X1        ; P = A1i*X1
      LTA       A2i      ; ACC = ACC + A1i*X1
      MPY       X2        ; P = A2i*X2
      LTA       A3i      ; ACC = ACC + A2i*X2
      MPY       X3        ; P = A3i*X3
      APAC                      ; ACC = ACC + A3i*X3
      ADDS      Round     ; Round result.
      ADDH      Sat       ; Saturate Y to Q10 value
      SUBH      Sat
      ADDH      Sat
      SACH      Y,1       ; Y = Q10 number.

; Cycles = 13 + 4n cycles (n = number of taps).

; Note: If saturation is not required, Cycles = 8 + 4n cycles
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Figure 1.

If the number of taps is greater than 6, then a RPT loop can be used for each bank and the effective cycles/tap can be approximately 2.

The above technique is almost equivalent to a floating-point notation with a 4-bit exponent and a 16-bit mantissa.